Air filtration simulation with focus on slip effects

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What's new?

Filtration community is interested in slip effects due to very small scales:

- Nano particles
- Nano fibers

Our earlier description [Latz & Wiegmann, Filtech 2003] *was not valid for these regimes*

- For particles, implemented **Cunningham correction**
- For fibers, **fractional slip** replaces no-slip boundary conditions

What's not so new?

'Physicists' model produces individual contributions of standard modeled effects:

- Interception
- Inertial Impaction
- Brownian Motion (Diffusion: Smoluchowski-Einstein-Langevin)
- Sieving (not today)

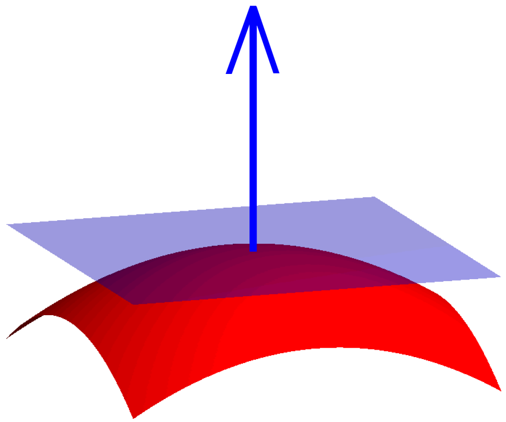
… but we never explained it that well!

Outline:

I. Mathematical models II. Results III. Discussion

No slip vs fractional slip

$$\begin{aligned} -\mu\Delta\vec{u} + \nabla p &= 0 \text{ (momentum balance)} \\ \nabla \cdot \vec{u} &= 0 \text{ (mass conservation)} \\ \vec{u} &= 0 \text{ on } \Gamma \text{ (no-slip on fiber surfaces)} \\ P_{in} &= P_{out} + c \text{ (pressure drop is given)} \end{aligned}$$



μ : fluid viscosity,

$\vec{u}$: velocity, periodic,

p : pressure, periodic up to pressure drop in flow direction.

$$\begin{aligned} -\mu\Delta\vec{u} + \nabla p &= 0 \text{ (momentum balance)} \\ \nabla \cdot \vec{u} &= 0 \text{ (mass conservation)} \\ \vec{n} \cdot \vec{u} &= 0 \text{ on } \Gamma \text{ (no flow into fibers)} \\ \vec{t} \cdot \vec{u} &= -\lambda \vec{n} \cdot \nabla \left(\vec{u} \cdot \vec{t} \right) \text{ on } \Gamma \text{ (slip flow along fibers)} \\ P_{in} &= P_{out} + c \text{ (pressure drop is given)} \end{aligned}$$

$\vec{n}$: normal direction to the fiber surface,

λ : slip length,

$\vec{t}$: any tangential direction with $\vec{t} \cdot \vec{n} = 0$.

Particle motion

$$d\vec{v} = -\gamma \times \left(\vec{v}(\vec{x}(t)) - \vec{u}(\vec{x}(t)) \right) dt + \sigma \times d\vec{W}(t),$$

$$d\vec{x} = \vec{v}(\vec{x}(t)) dt,$$

$$\sigma^2 = \frac{2 k_B T \gamma}{m},$$

$$\gamma = 6\pi\rho\mu \frac{R}{m},$$

$$\langle dW_i(t), dW_j(t) \rangle = \delta_{ij} dt.$$

$\vec{u}$: fluid velocity

$\vec{v}$: particle velocity

t : time

T : temperature

γ : friction coefficient

R : particle radius

ρ : fluid density

μ : fluid viscosity

$\vec{W}$: Wiener Measure (3d)

k_b : Boltzmann constant

m : particle mass

Kn : Knudsen number

[Latz & Wiegmann, Filtech 2003]

Interception

$$d\vec{v} = -\gamma \times \left(\vec{v}(\vec{x}(t)) - \vec{u}(\vec{x}(t))\right) dt + \sigma \times d\vec{W}(t),$$

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$$m \to 0, \sigma = 0$$

$$\Rightarrow \vec{v} = \vec{u}$$

$$\Rightarrow d\vec{x} = \vec{u}(\vec{x}(t))dt.$$

Disable diffusion and let mass tend to zero (keeping all other parameters fixed), the particle velocity tends to the fluid velocity.

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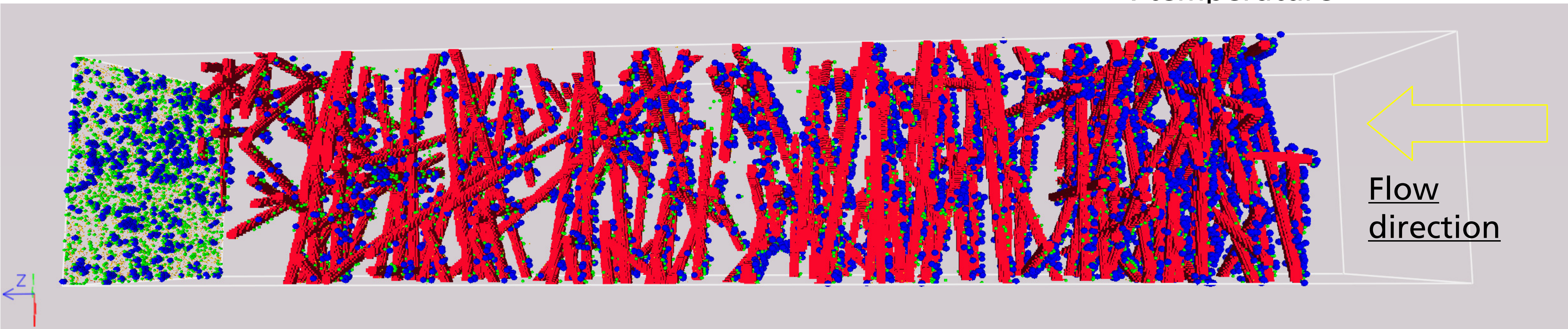

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Inertial Impaction

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By only switching off diffusion, the effect of inertial impaction is the difference between this simulation and the one for vanishing mass (interception).

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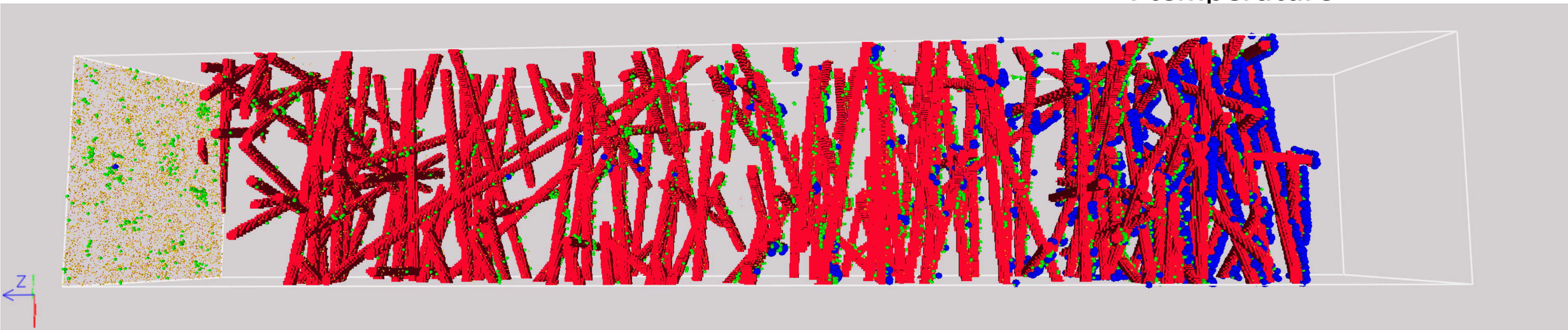

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Particle Motion including Cunningham correction

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$$\sigma^2 = \frac{2k_B T \gamma}{m},$$

$$\gamma = 6\pi\rho\mu \frac{R}{C_c m},$$

$$\langle dW_i(t), dW_j(t) \rangle = \delta_{ij} dt,$$

$$C_c = 1 + Kn\left[1.142 + 0.558 e^{-0.999/Kn}\right],$$

$$Kn = \frac{\lambda}{R},$$

$$\lambda = \frac{k_B T}{\sqrt{32}\pi R^2 P}.$$

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- $\vec{v}$: particle velocity
- t : time
- T : temperature
- γ : friction coefficient
- R : particle radius
- ρ : fluid density
- μ : fluid viscosity
- $\vec{W}$: Wiener Measure (3d)
- k_b : Boltzmann constant
- m : particle mass
- Kn : Knudsen number
- : Total pressure

As *Kn* increases, influence of friction γ and diffusion σ decrease: particles *go more straight.*

Cunningham correction is always used later, also for interception and inertial impaction.

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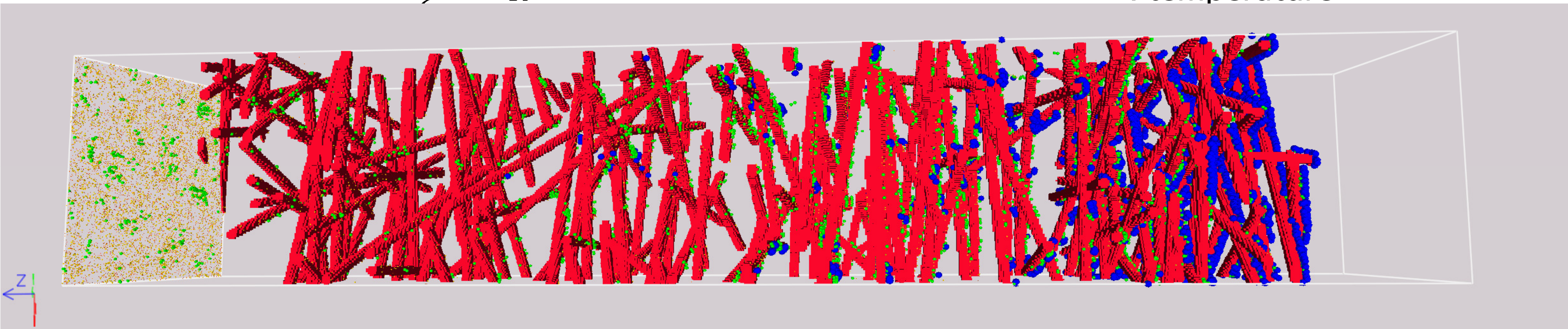

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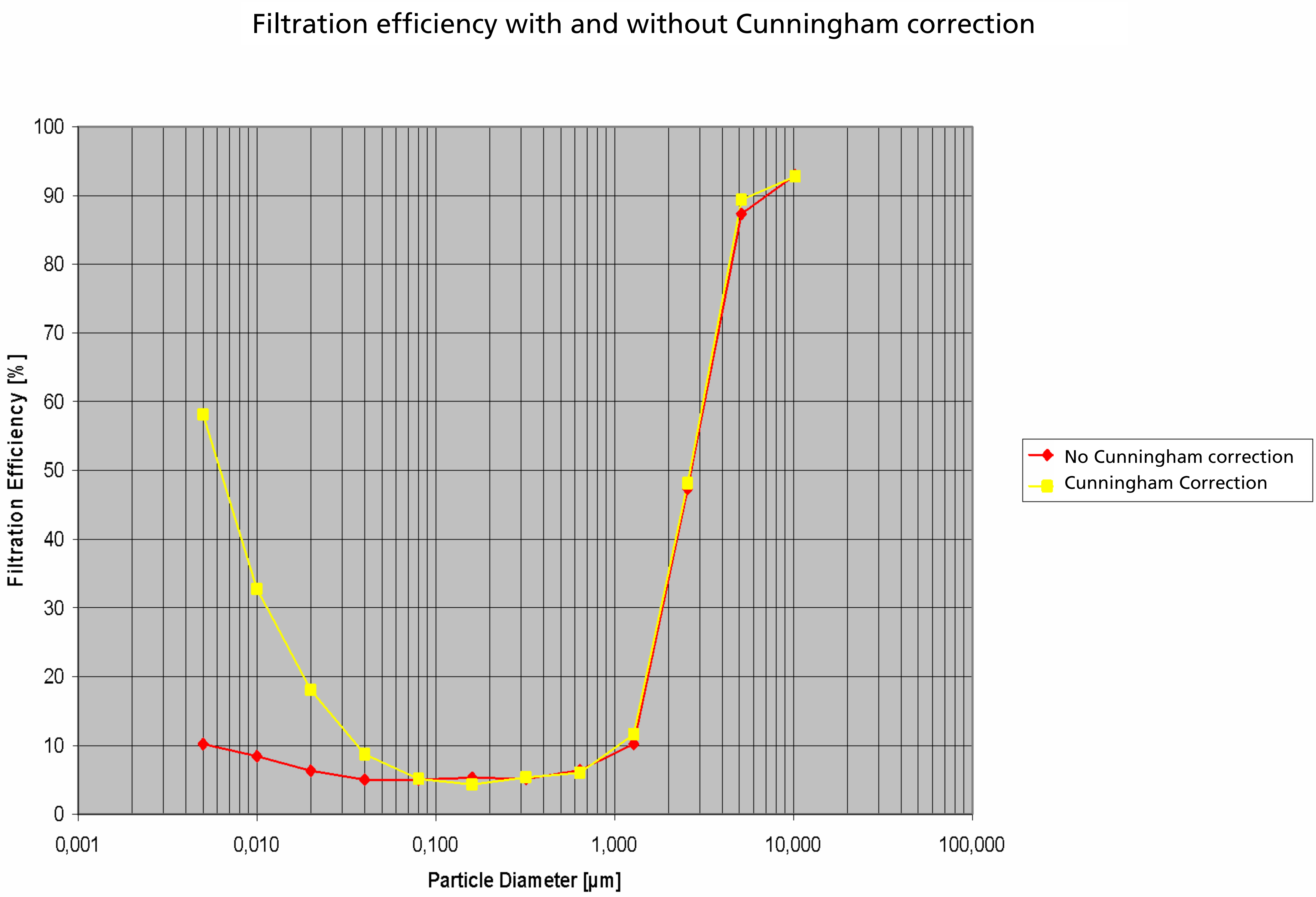
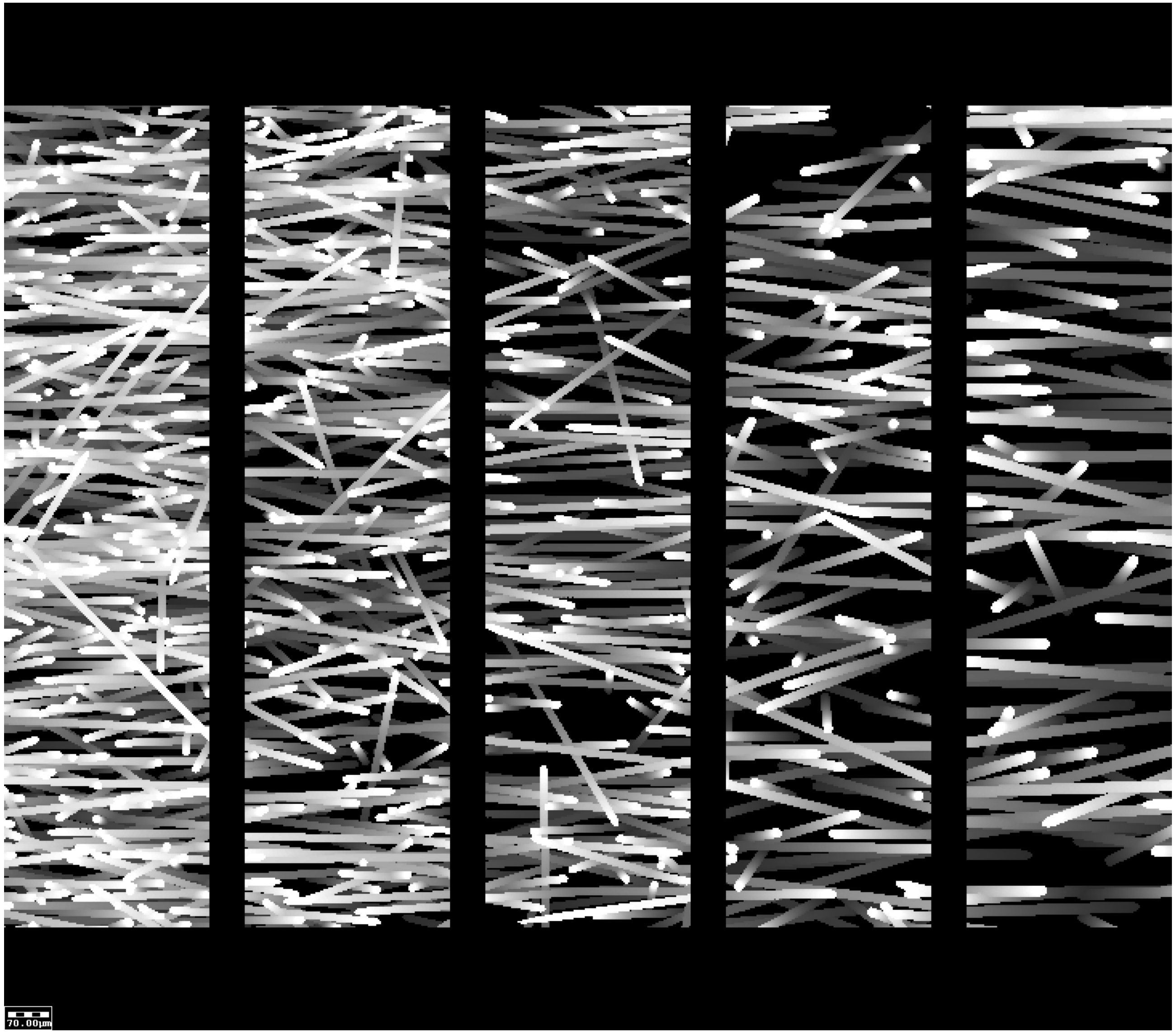


Filtration efficiency with and without Cunningham correction
Filtration Efficiency [%]
100
90
80
70
60
50
40
30
20
10
0
0,001
0,010
0,100
1,000
10,000
100,000
Particle Diameter [µm]
No Cunningham correction
Cunningham Correction

Simulated SEM view of 5 Structures



$\alpha = 0.1$	$\alpha = 0.07$	$\alpha = 0.05$	$\alpha = 0.05$	$\alpha = 0.05$
$d_F = 14\mu m$	$d_F = 14\mu m$	$d_F = 14\mu m$	$d_F = 17\mu m$	$d_F = 20\mu m$

α: solid volume fraction of fibers

d_F: fiber diameter

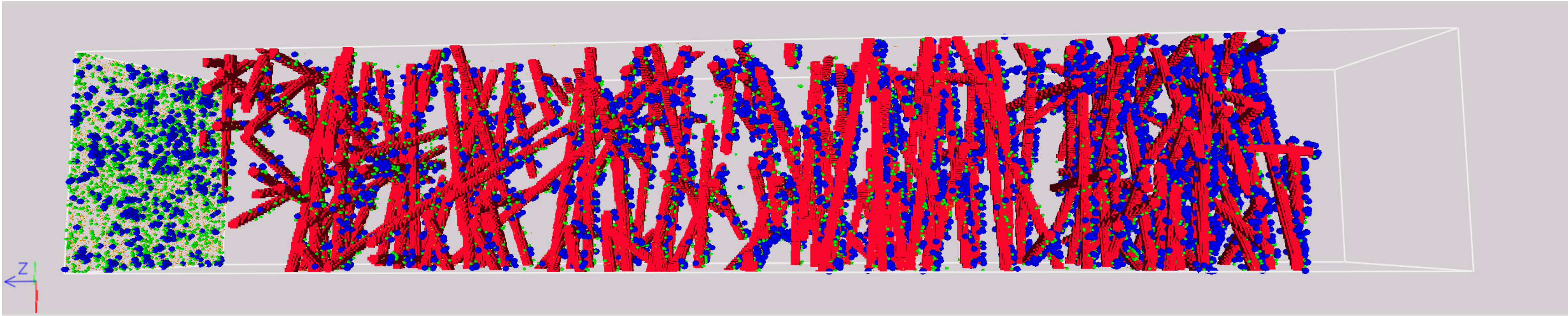
Deposition effects

α = 0.05,

d_F = 14,

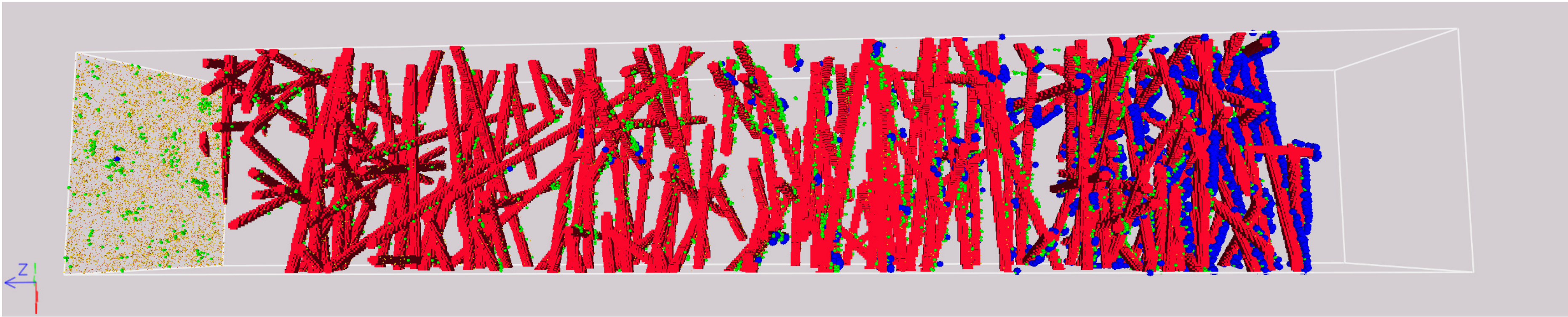
v = 0.1m/s

Interception



Interception

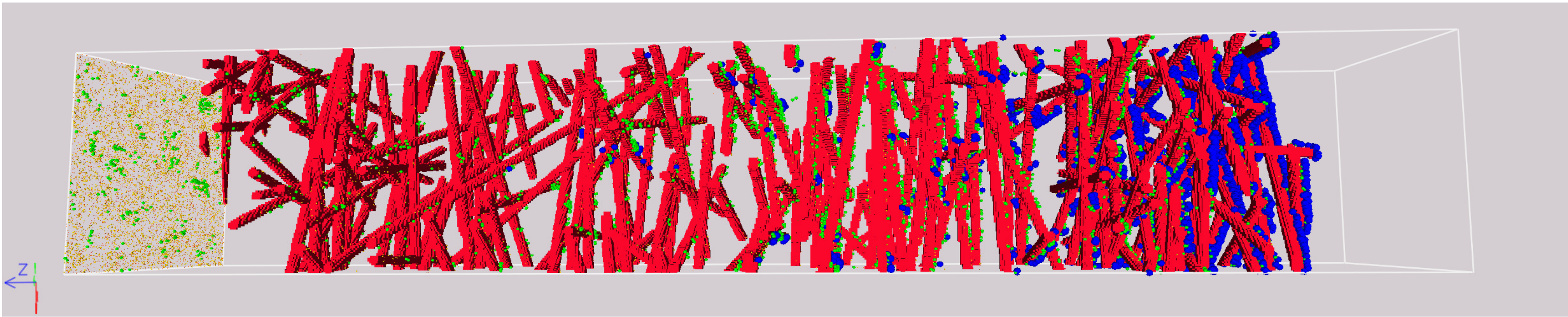
+ Impaction



Interception

+ Impaction

+ Diffusion



Velocity Effects

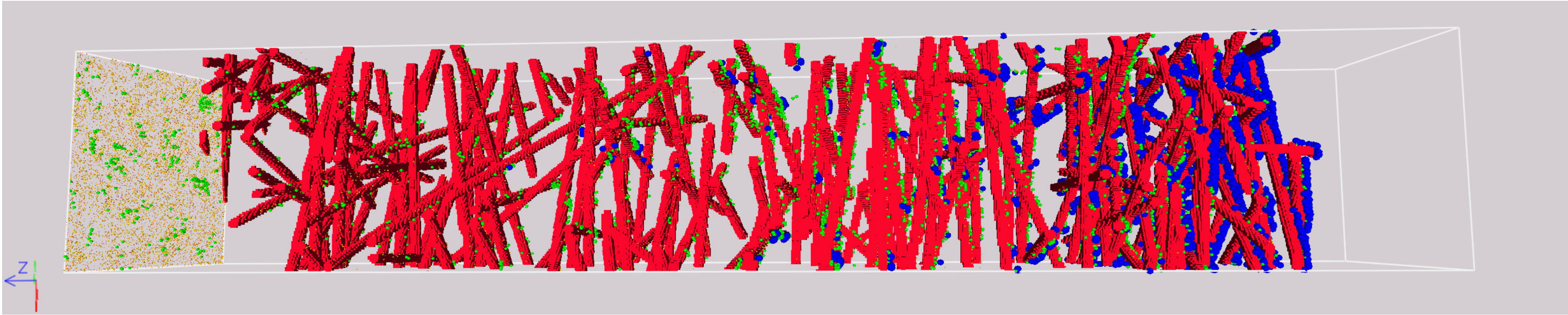
α = 0.05,

dF = 14,

Interception + Impaction + Diffusion

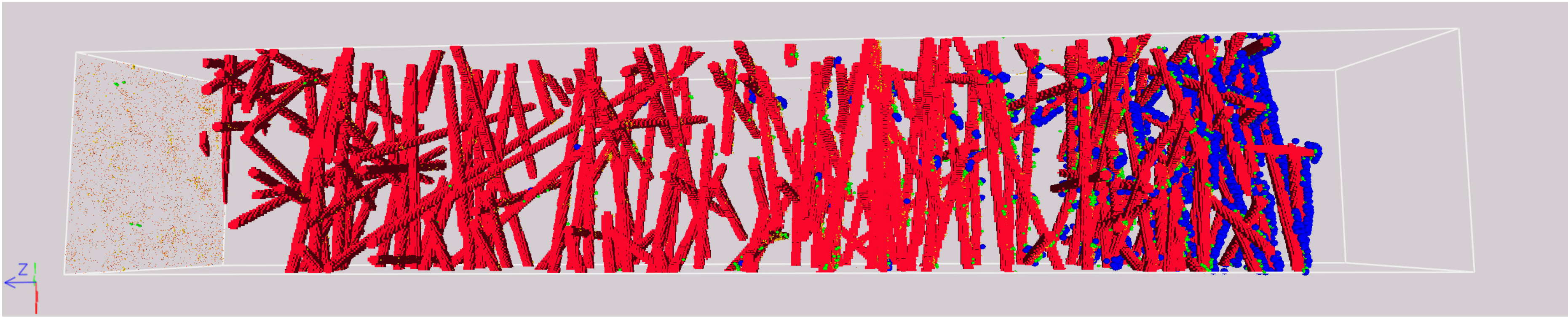
Velocity

v = 0.1 m/s



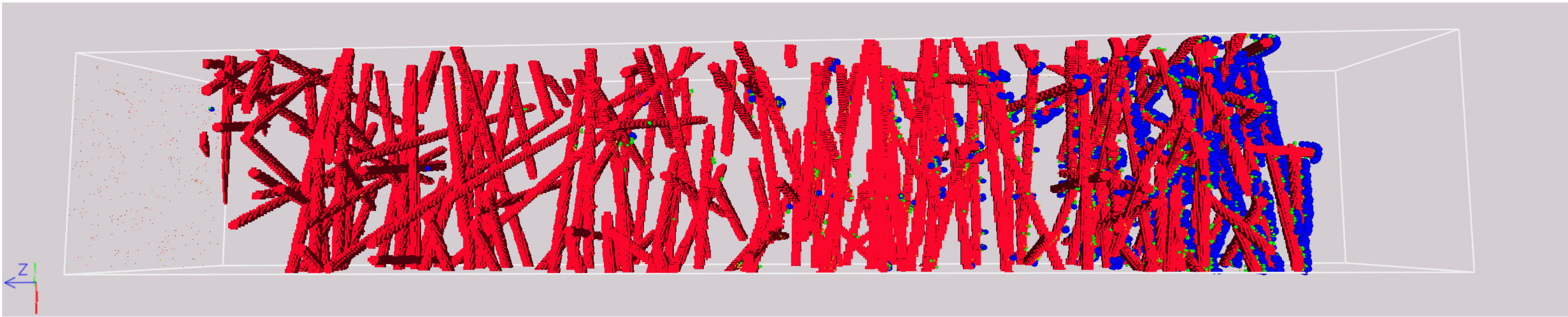
Velocity

v = 1 m/s



Velocity

v= 10 m/s



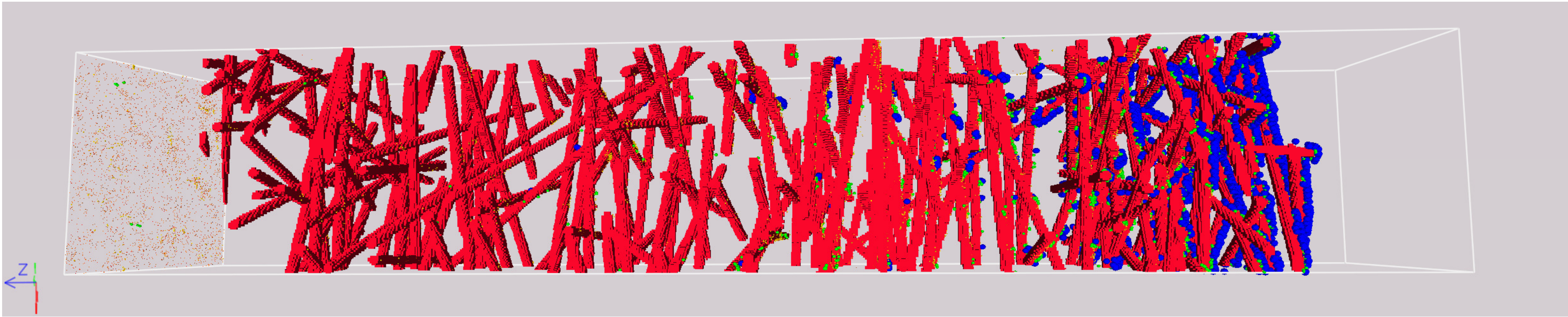
Effect of solid volume fraction (SVF)

dF = 14,

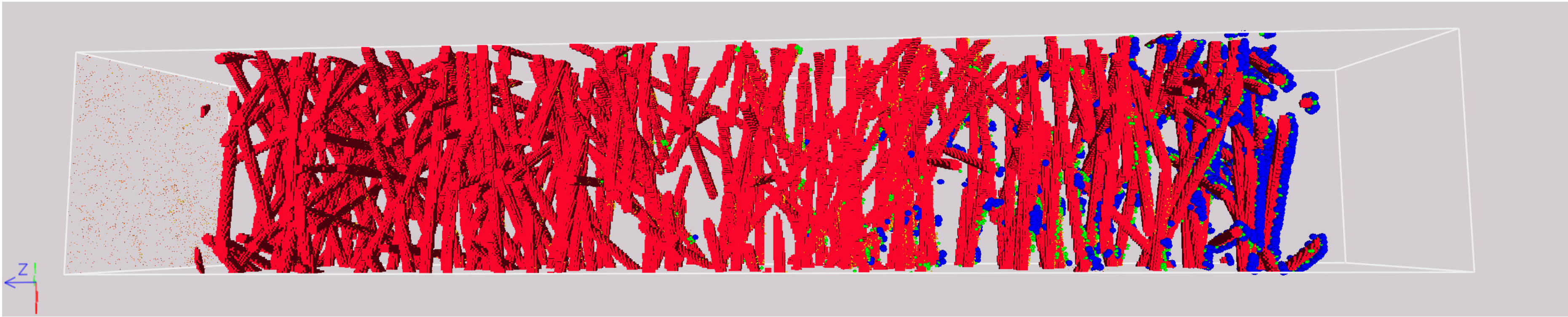
v = 1 m/s,

Interception + Impaction + Diffusion

SVF α=0.05



SVF α=0.07



SVF α=0.1

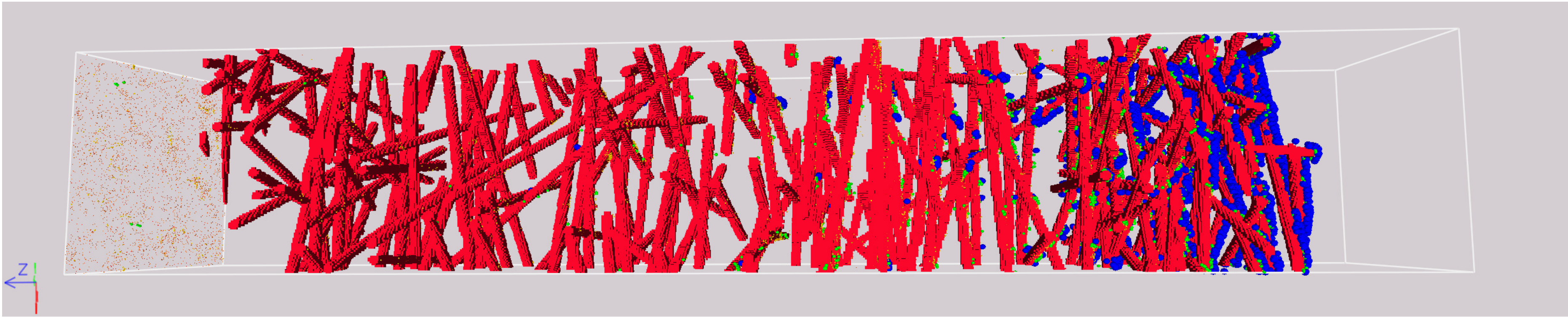
Effect of fiber thickness

$\alpha = 0.05$,

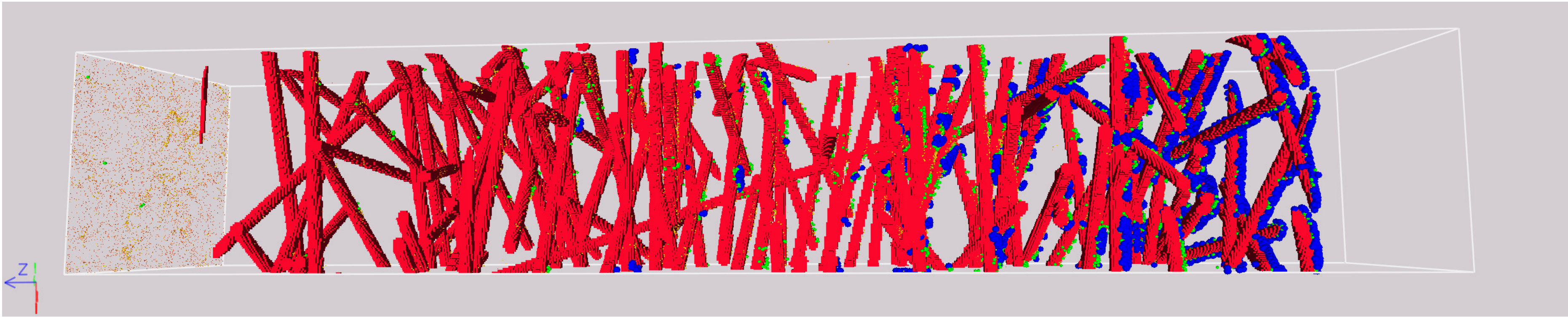
v = 1 m/s

Interception + Impaction + Diffusion

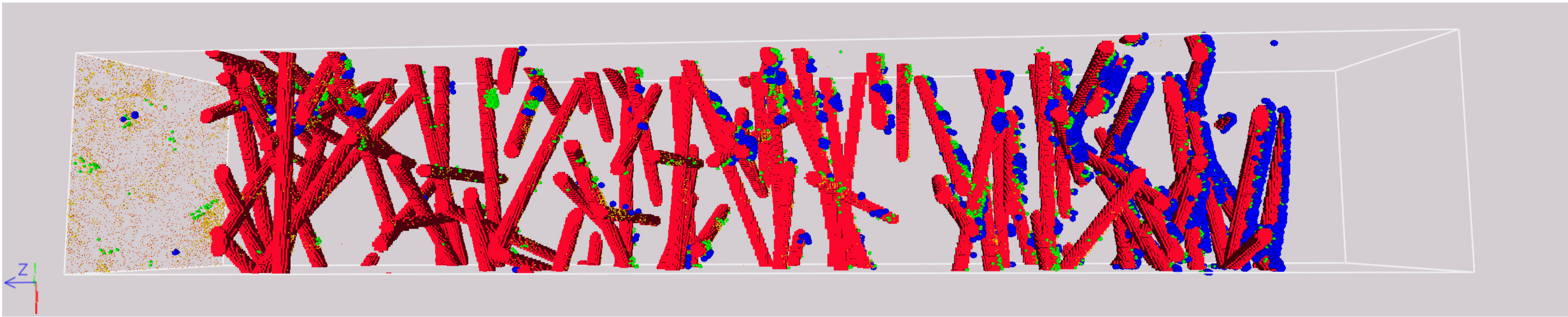
dF = 14



dF = 17



dF = 20



Effect of slip flow

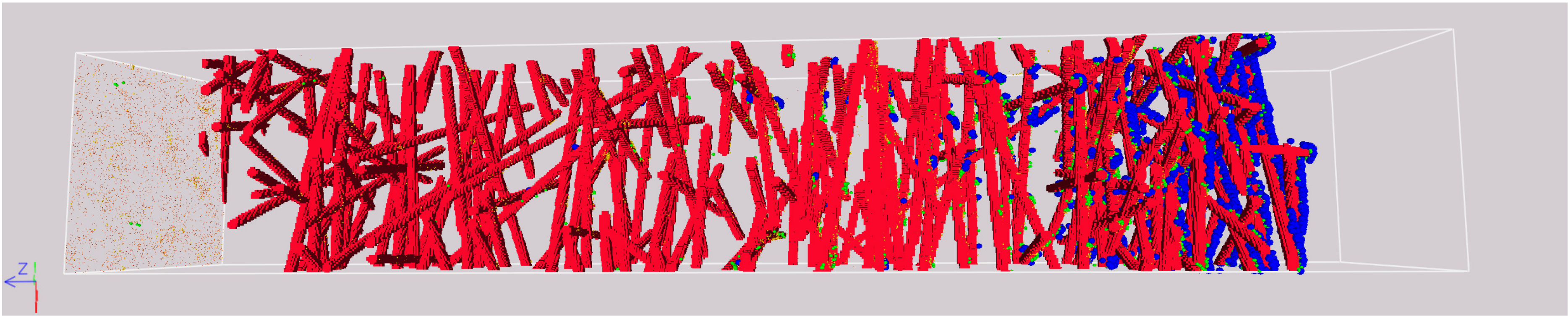
α= 0.05,

dF = 14,

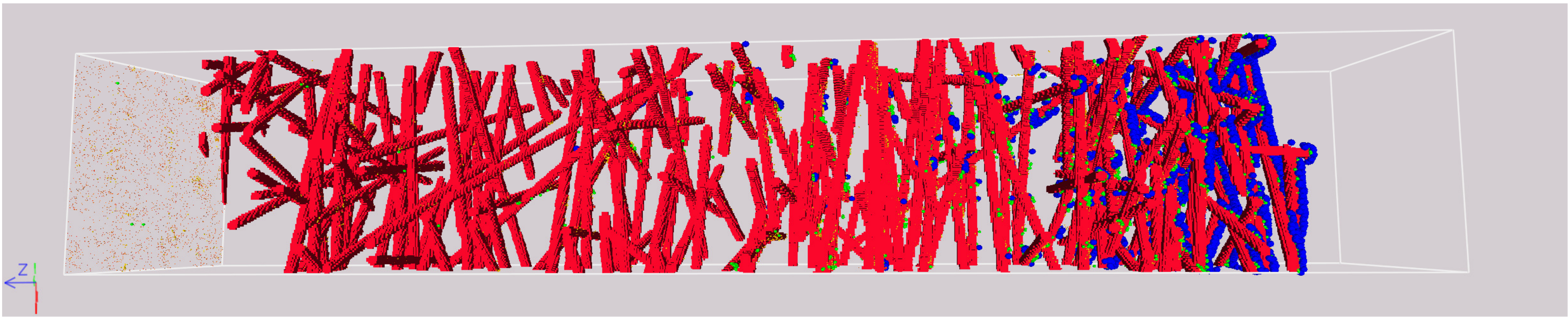
v = 1 m/s

Interception + Impaction + Diffusion

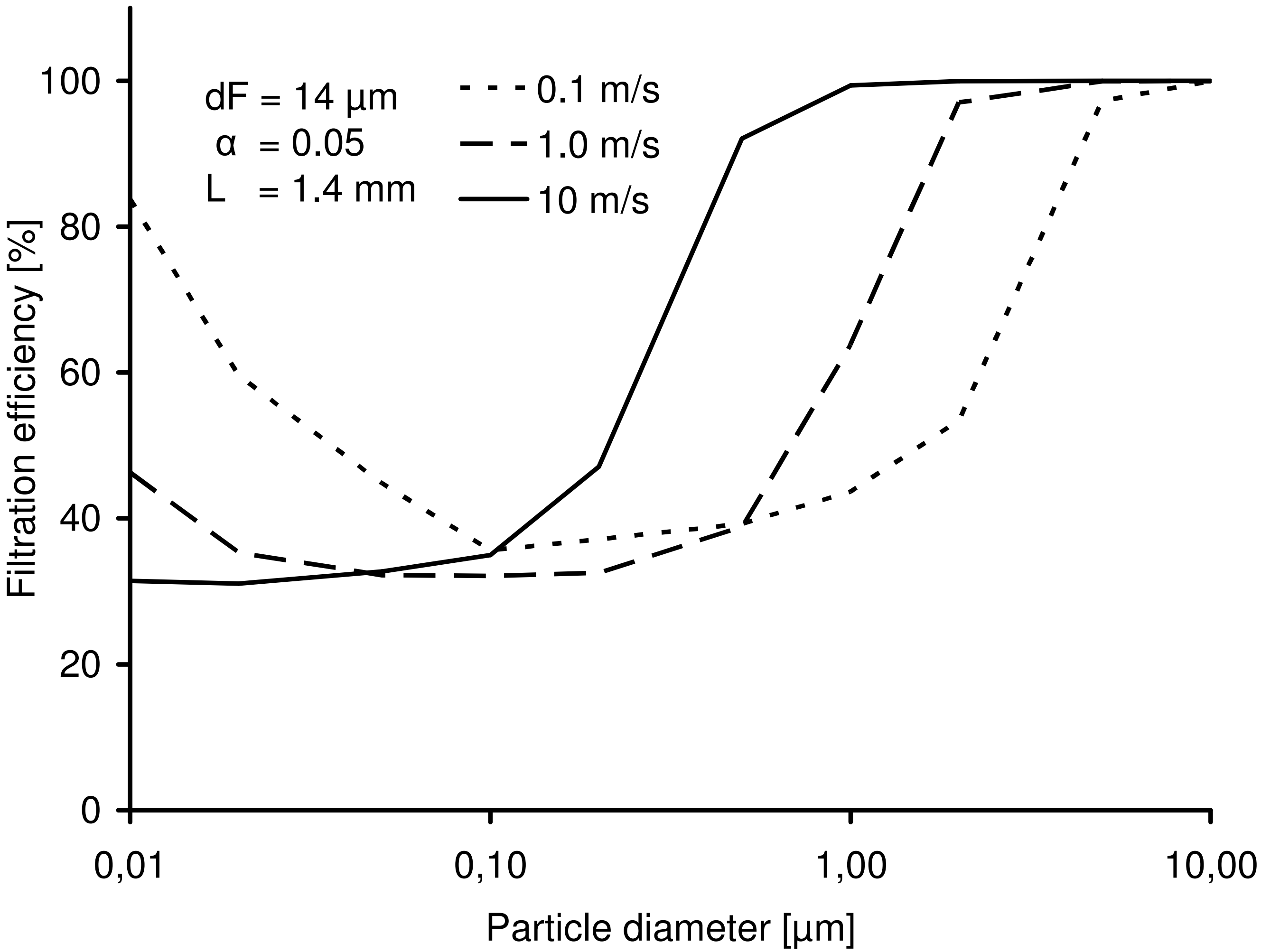
No slip boundary conditions



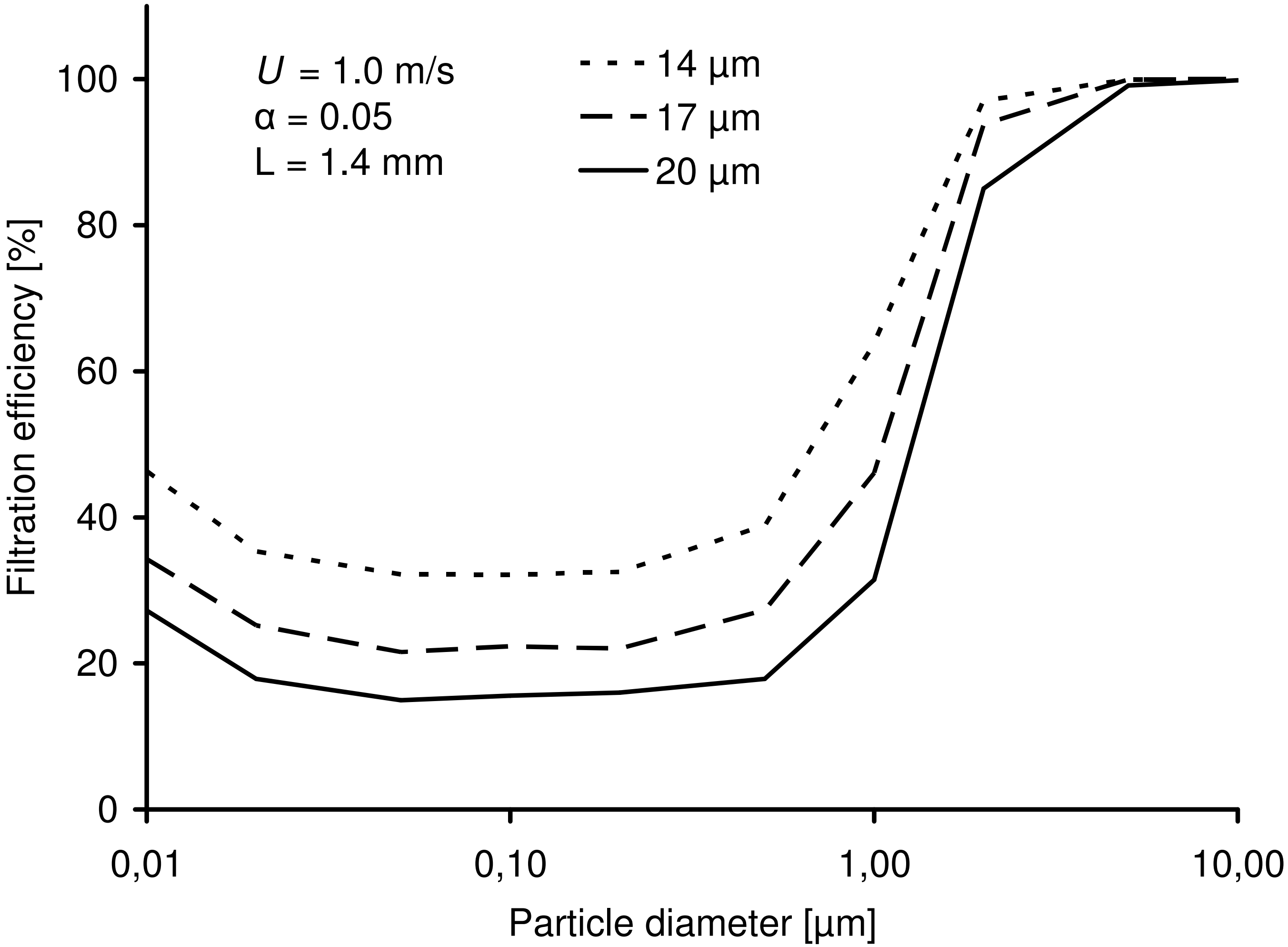
Fractional slip boundary conditions



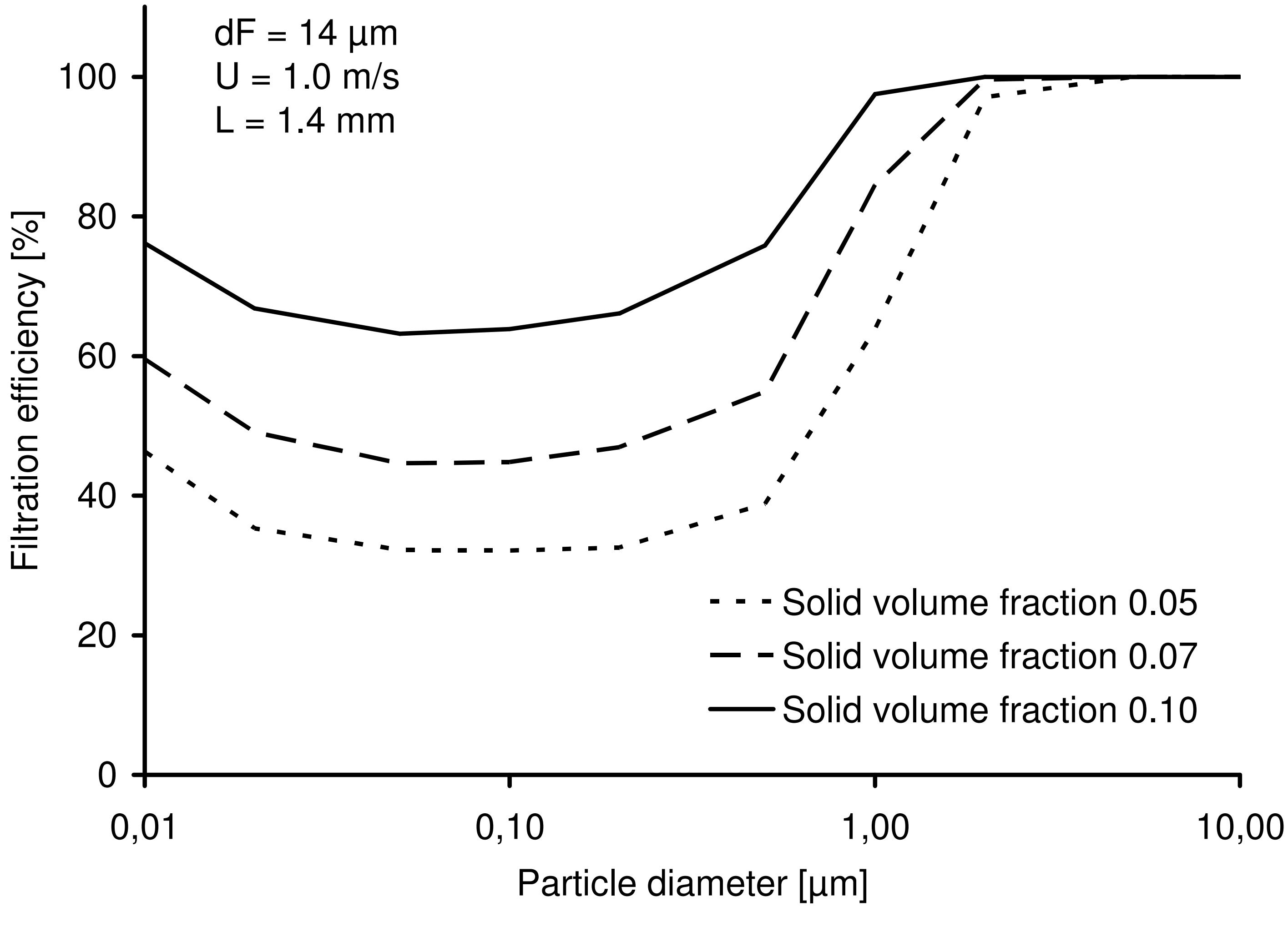
Influence of velocity on filter efficiency



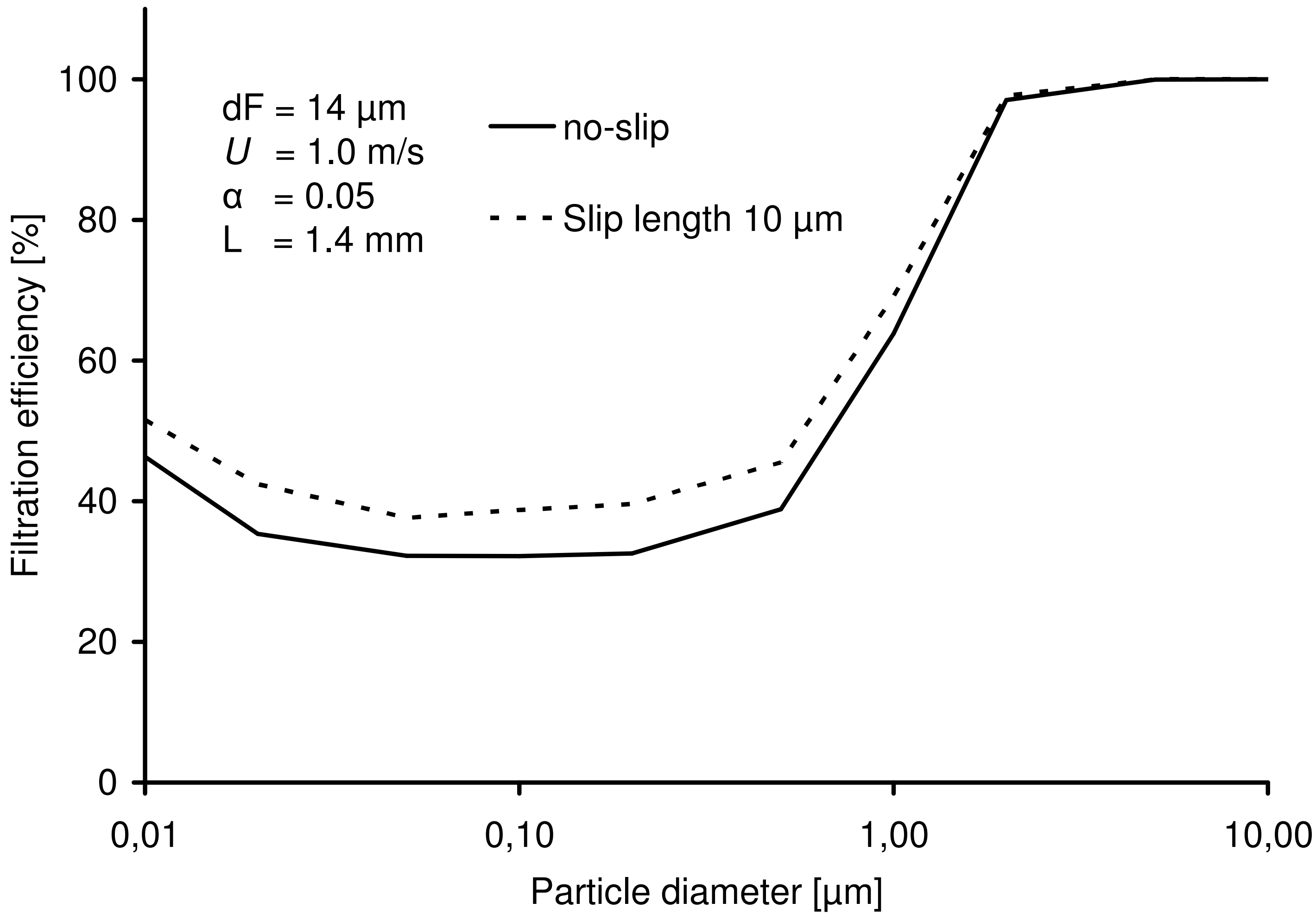
Influence of fiber diameter on filter efficiency



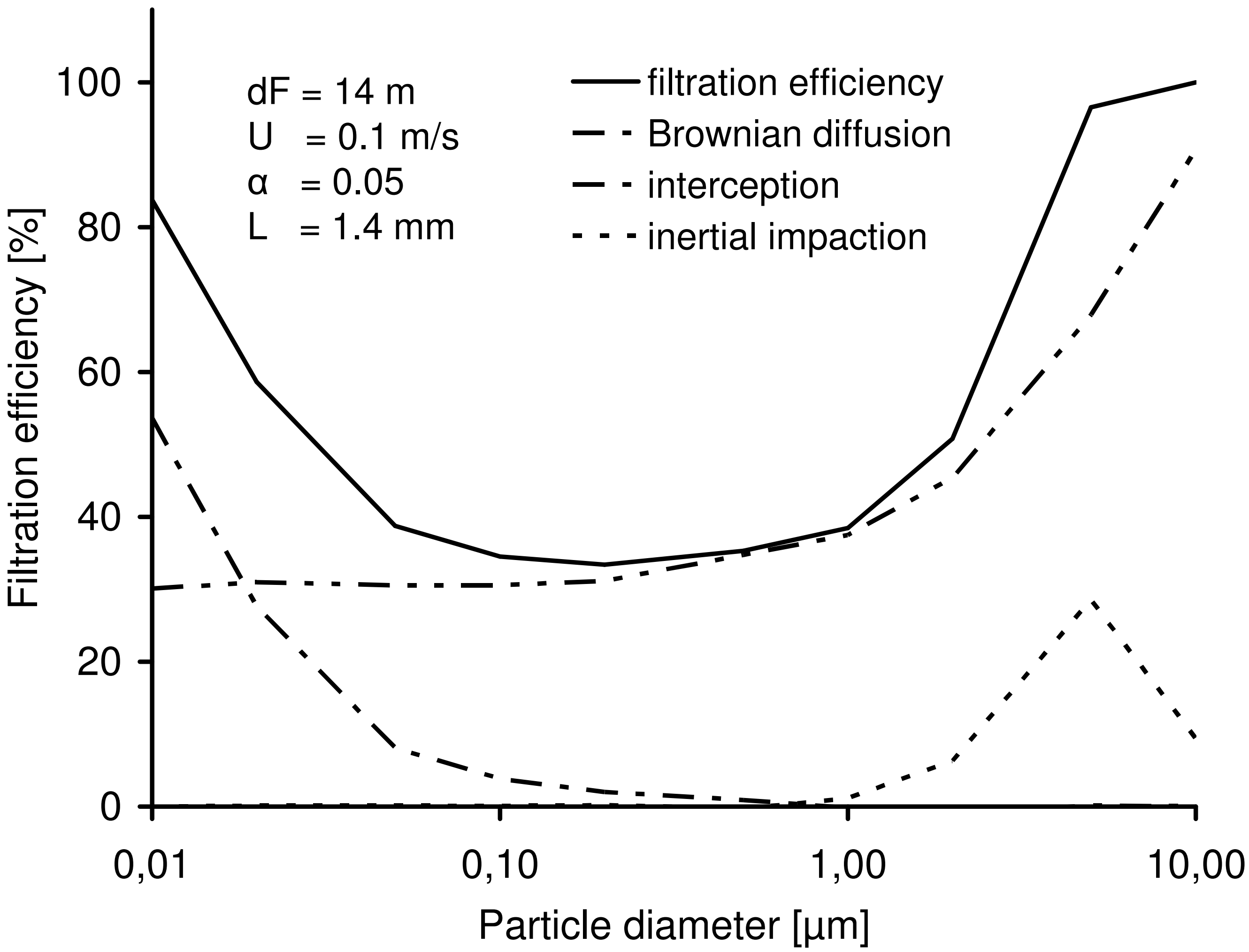
Influence of SVF on filter efficiency



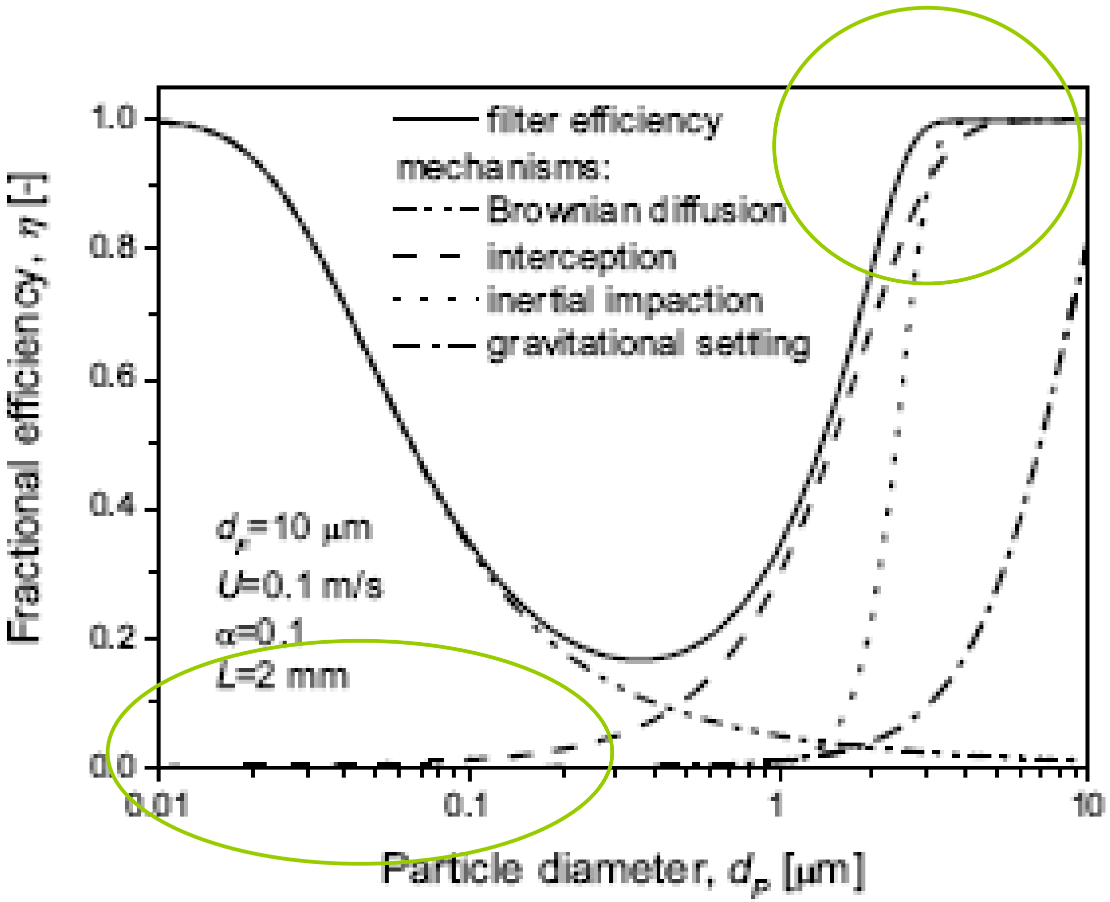
Influence of slip flow on filter efficiency



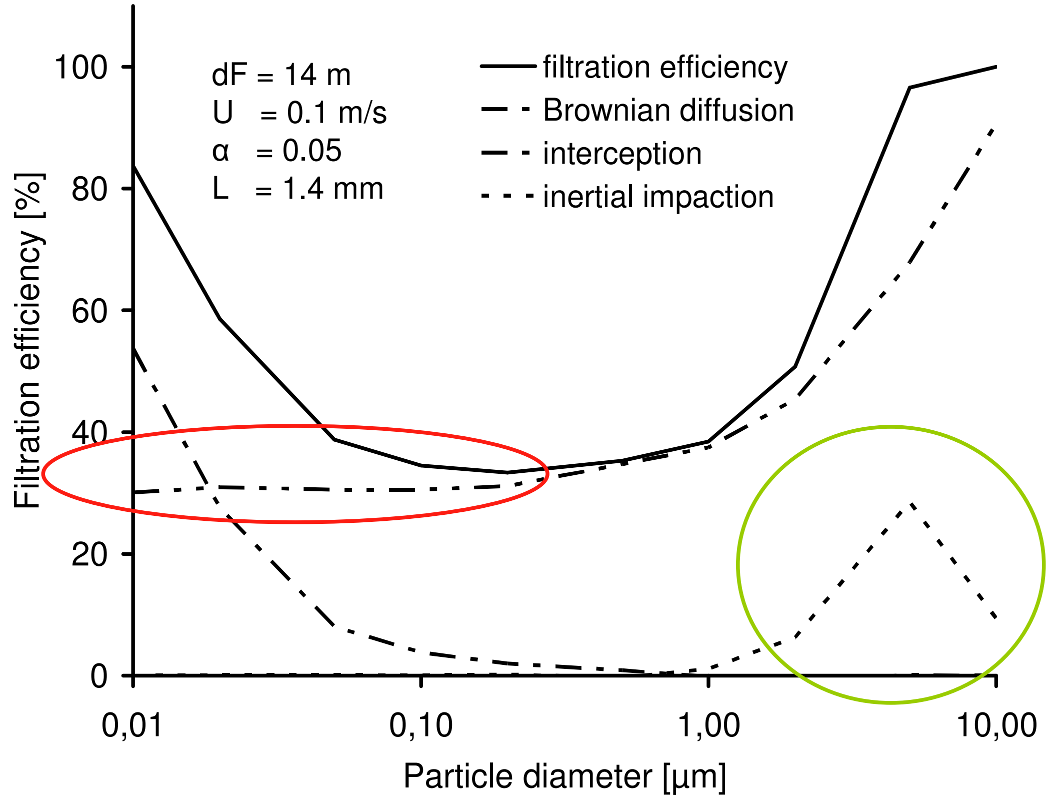
Influence of individual effects on filter efficiency



Comparison with [Balazy & Podgorski Filtech 2007]

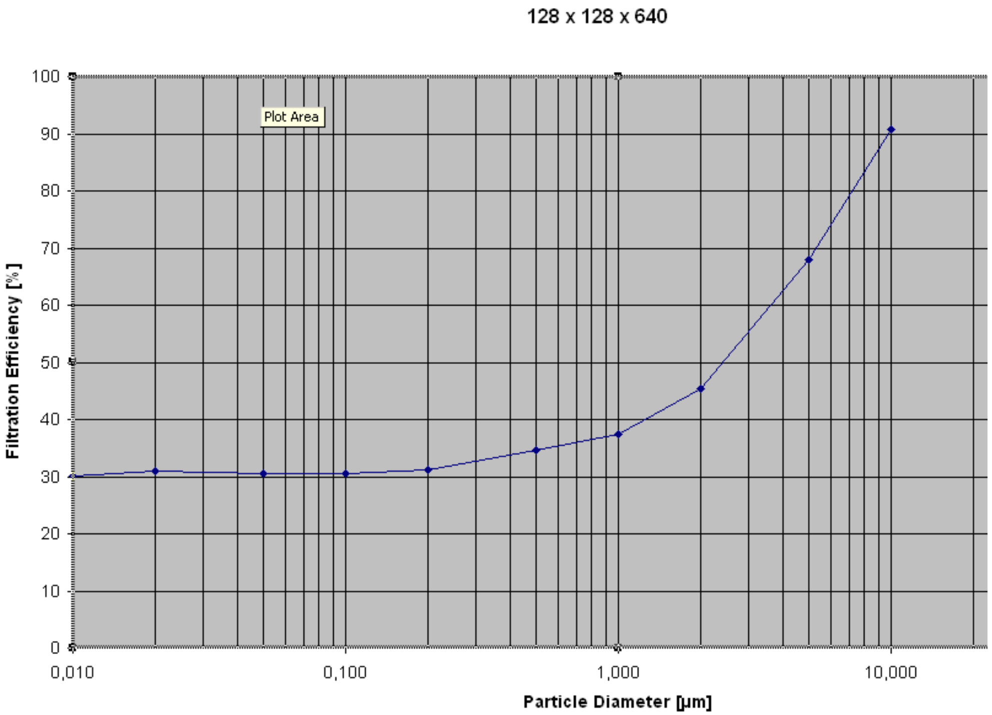


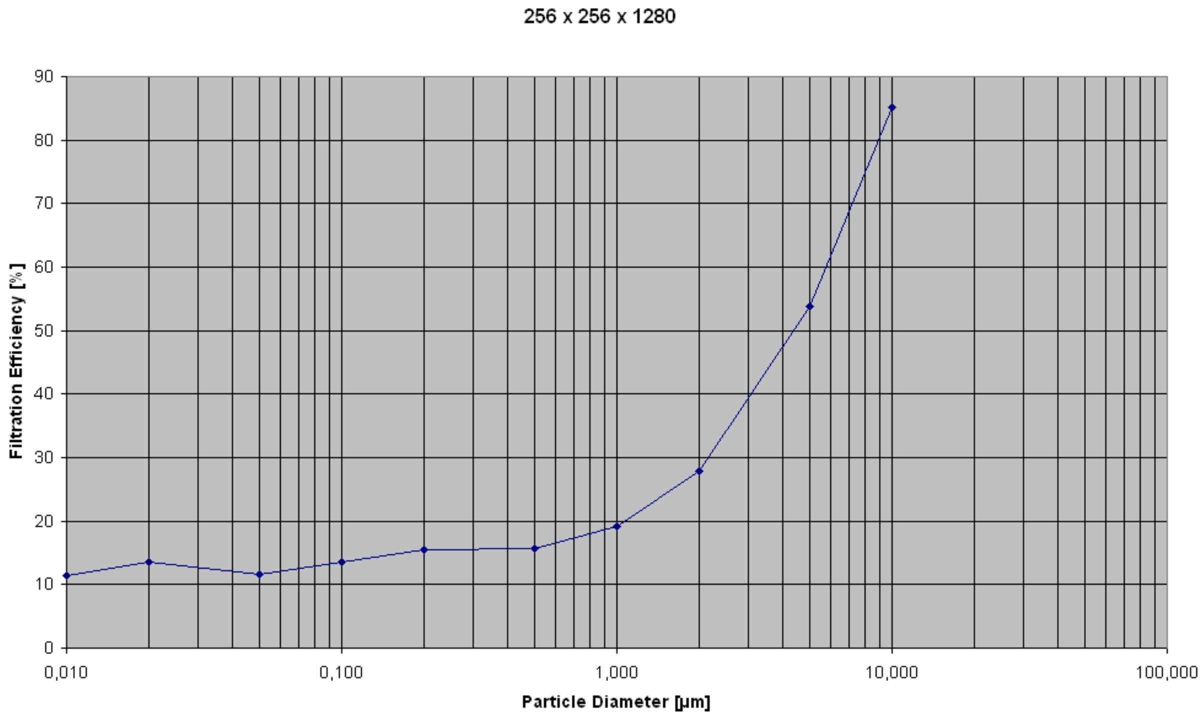
[Balazy & Podgorski Filtech 2007]



Current work

Effect of mesh refinement on efficiency due to interception





We believe [B &P] results for nano particles. We get their *trend* for more refined mesh and will investigate what causes the effect for the coarser resolution – the pressure drop is ok, difference hopefully lies in some details of the flow field, particle tracking, or particle collision computations…

Conclusions

Models for nanoscale effects for

- particles
- fibers

added to the model and implemented in the code

Parameters in ‘physicists model’ set to achieve ‘decomposition into classical effects’

- Interception
- Inertial Impaction
- Diffusion

Nano scale results agree qualitatively with literature and measurements, further work is in progress for quantitative agreement

Nonwoven models, computations and figures made with our Software

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Thank you for your time and attention