On Coupled Particle Level and Filter Element Level Simulation for Filtration Processes

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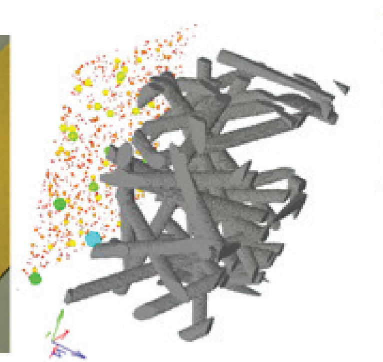

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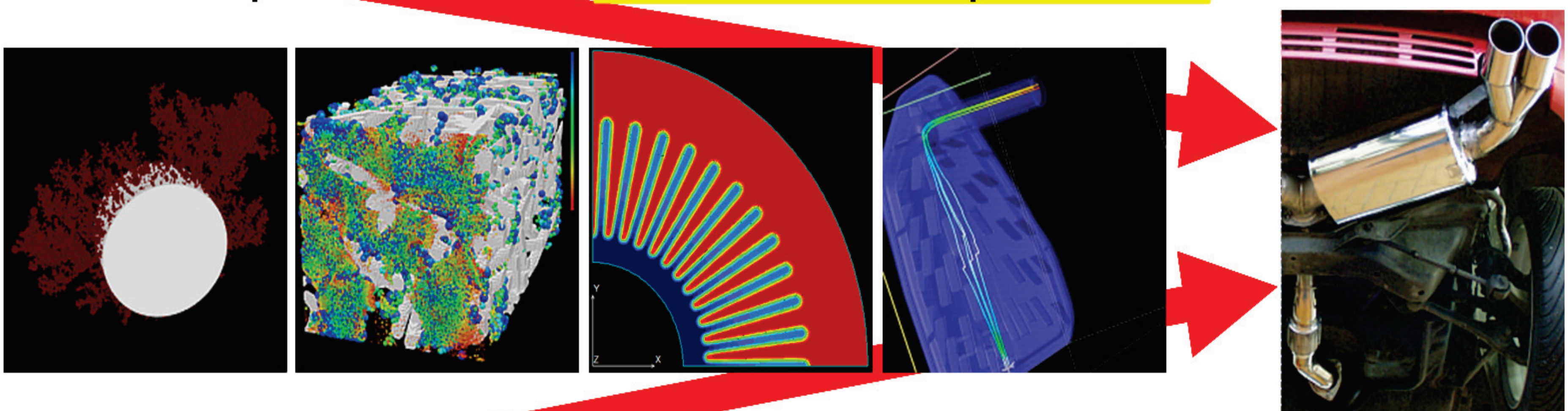
Multiple scales in filtration

Particles level	Filter components		Filter element	Complete system
Nano scale	Micro scale	Millimeter	Centimeter	Meter

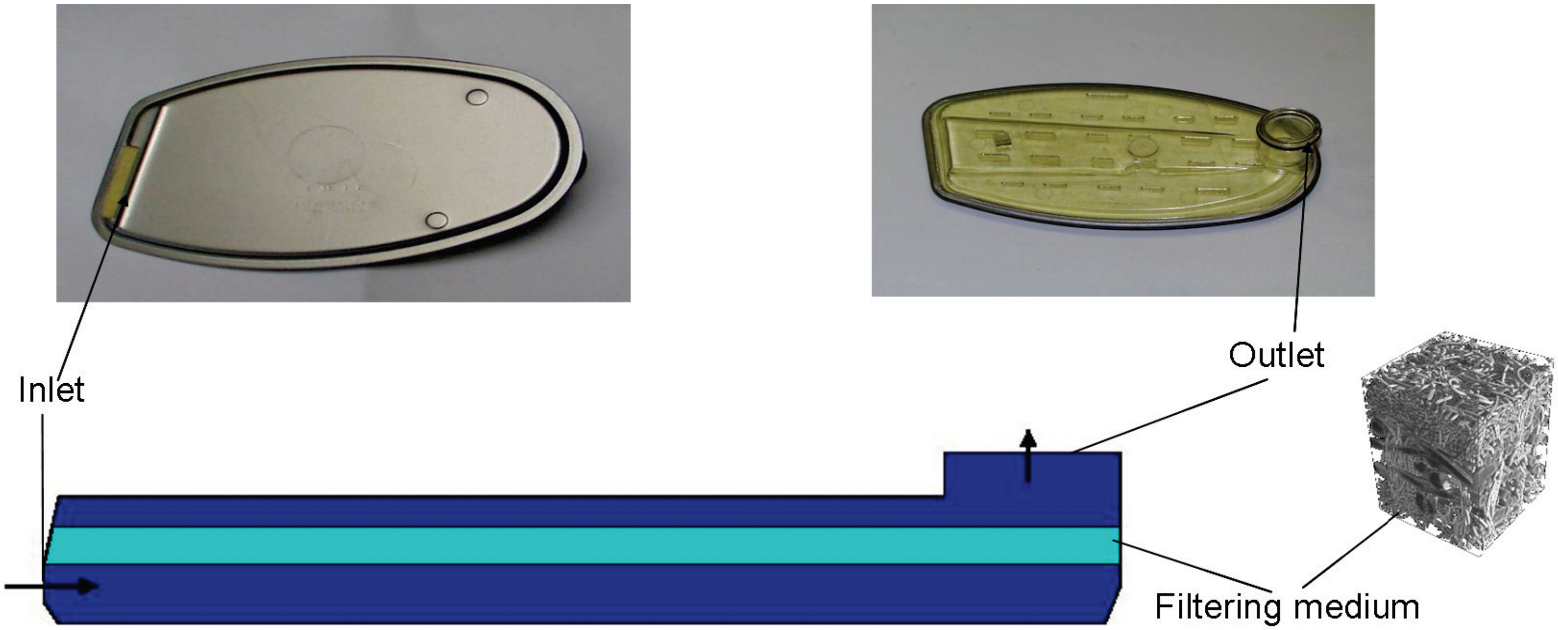
(Navier-)Stokes in pore space coupled with stochastic ODE for particles, ...

(Navier-)Stokes-Brinkmann in fluid and in porous media coupled with concentration of particles



Particles-Fiber interaction | Dirt loading of the filtering medium | Pleats in cartrige filters | Flow within Filter element | Filter installation

Multiple scales in filtration



Determining performance of a filter

- Pressure drop - flow rate ratio
- Dirt Storage capacity
- Size of penetrating particles

Depend on microscale (e.g. fibrous geometry) and macroscale (e.g. pressure, velocity) quantities.

Navier-Stokes-Brinkmann System

Incompressible laminar flow through filters

$$\underbrace{\frac{\partial \vec{u}}{\partial t} - \nabla \cdot (\tilde{\mu} \nabla \vec{u}) + (\rho \vec{u}, \nabla)\vec{u}}_{Navier-Stokes} \overbrace{+\mu \tilde{K}^{-1} \vec{u} + \underbrace{\nabla p = f}}^{Darcy}$$

$$\nabla \cdot \vec{u} = 0 \qquad \text{continuity equation}$$

Fictitous Region Method

Type continuation of coefficients:

$$\tilde{K}^{-1} = \begin{cases} K^{-1}, \; x \in \Omega_p \\ 0, \;\; x \in \Omega_f \end{cases}$$

Interface conditions

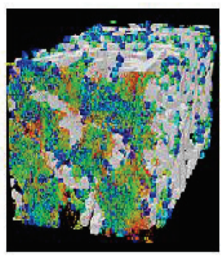
$$[\vec{u}]_\Gamma = 0$$

$$[\vec{n} \cdot (\tilde{\mu} \nabla \vec{u} - pI]_\Gamma = 0$$

Justification in Angot 1998

Macroscale model

- Estimate the pressure drop within the filter element
- Varying permeability (depending on loading)
- Missing: Information about the processes at the pore/particle level



Extension of macroscale model to account for particles' loading

- Solve the convection-diffusion-reaction equation at the meso scale.

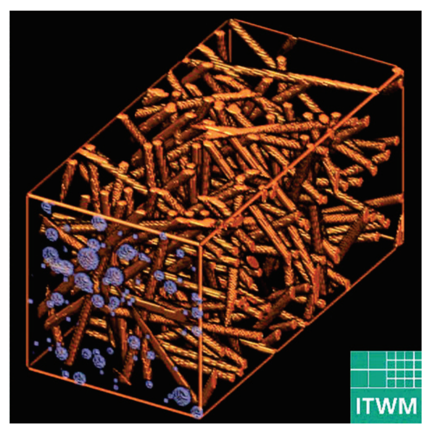
Convection Diffusion Reaction equation

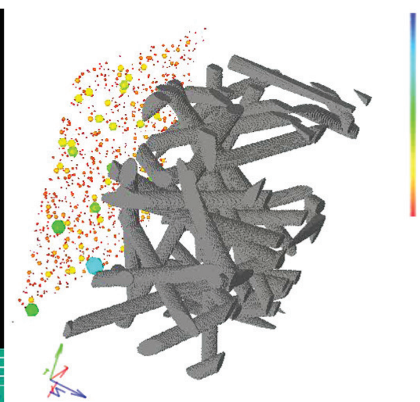
$$\frac{\partial C^J}{\partial t} + (\vec{u}, \nabla C^J) - D\Delta C^J = \frac{\partial M}{\partial t}$$
$$\frac{\partial M}{\partial t} = -\alpha^J C^J$$

J - particle size
t - time
u - velocity
C - concentration
D - diffusion coefficient
$\alpha(t, u, C)$ - Absorption Rate

α derived from measurements or directly from microscale simulations

t	β
1	51.9
3	39.0
5	33.2
7	25.3
9	10.9





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Stokes system

$$\mu \Delta \vec{u} + \vec{f} = \nabla p$$
$$\nabla \cdot \vec{u} = 0$$

t - time
$\vec{u}$ - fluid velocity
p - pressure
μ - fluid viscosity

Stochastic ODE

$$\frac{\partial \vec{u_0}}{\partial t} = -\gamma \times (\vec{u_0}(\vec{x}) - \vec{u}(\vec{x})) + \frac{Q\vec{E}(\vec{x})}{m} + \sigma \times \frac{d\vec{W}(t)}{dt}$$
$$\frac{\partial \vec{x}}{\partial t} = \vec{u_0}$$
$$\sigma^2 = \frac{2k_B T\gamma}{m}$$
$$\gamma = 6\pi\rho\mu\frac{R}{m}$$
$$\left\langle d\vec{W}_i(t), d\vec{W}_i(t)\right\rangle = \delta_{ij} dt$$

$\vec{x}$ - particle position
$\vec{u_0}$ - particle velocity
m - particle mass
Q - particle charge
E - electric field
$d\vec{W}(t)$ - 3D probability measure
ρ - fluid density
R - particle radius

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Macroscopic scale

- Used for simulating filtration processes at filter element level
- Needs measurements at element level to calculate deposition rate and change in permeability
- Does not provide info about particles at pore/particle level

Microscopic scale

- Limited computer power
- Good for local simulations
- Unable to solve at level of filter element due to computer power limitations.

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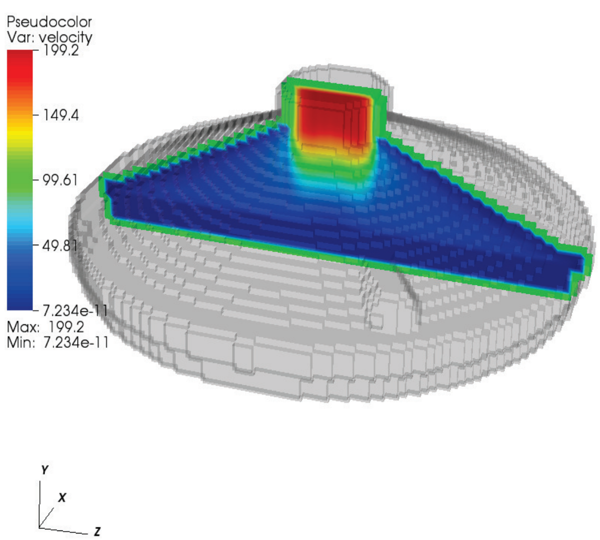
Sketch of the algorithm

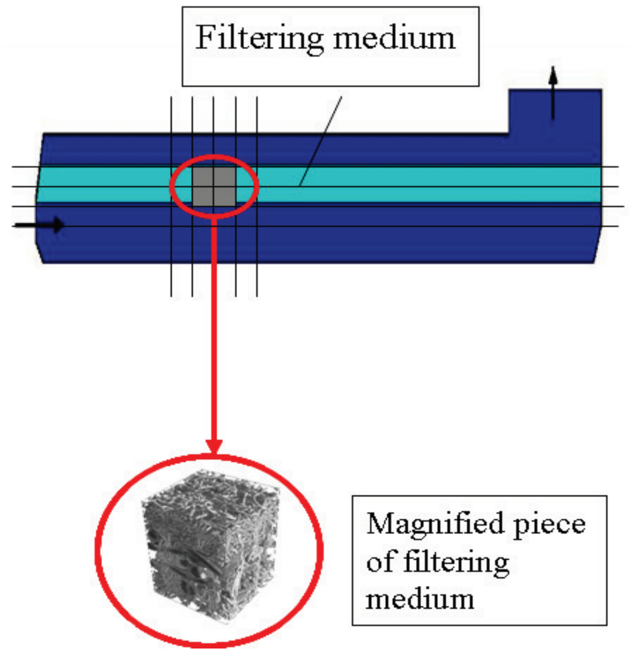
Step 1

Δp, $\vec{u}$ and C^J computed on the macro scale



Step 2

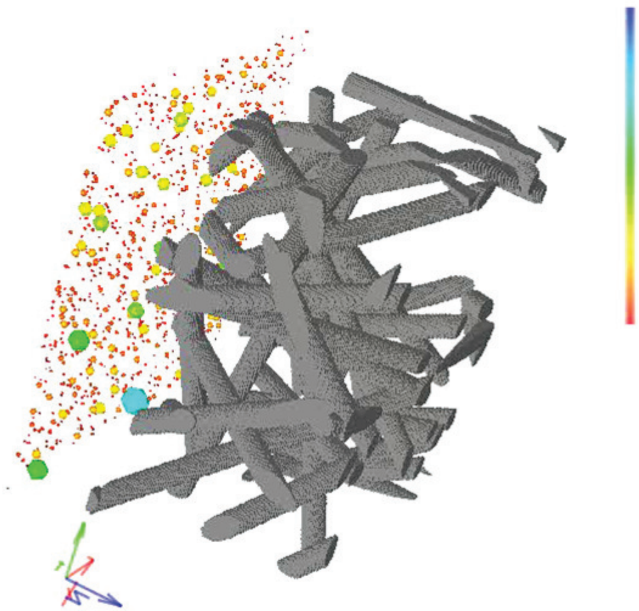
Downscaling of local velocity and particle concentrations on selected voxels



Sketch of the algorithm

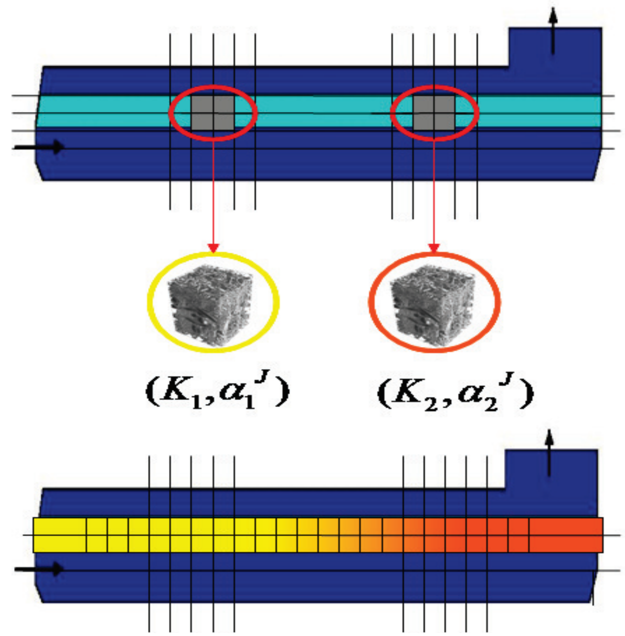
Step 3

Solve micro problem locally and compute new values for K and α

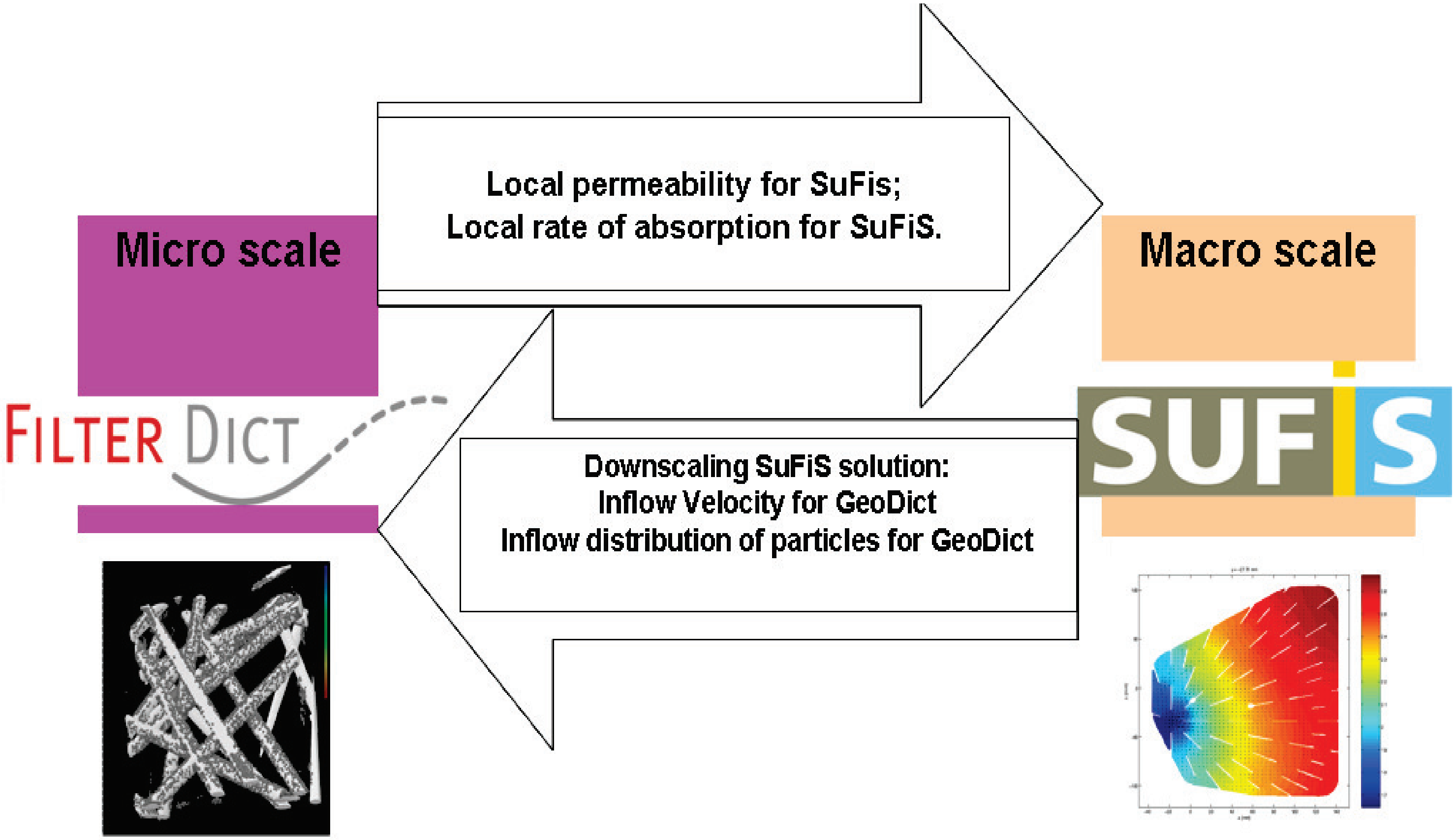


Step 4

Use interpolation techniques to compute α and K everywhere



Sketch of the algorithm



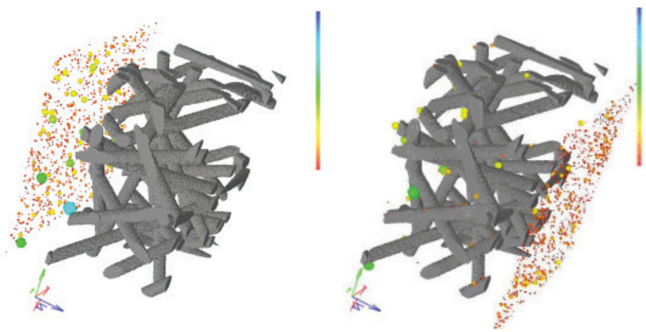
Details to be noted

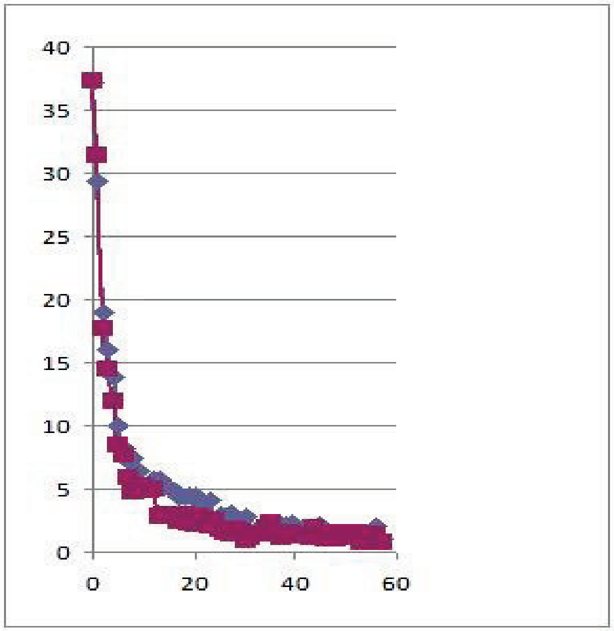
- different range of velocities
- change of particle concentration over time

Incorporate info in the form of correlation tables

Results

Correlation tables from micro simulations



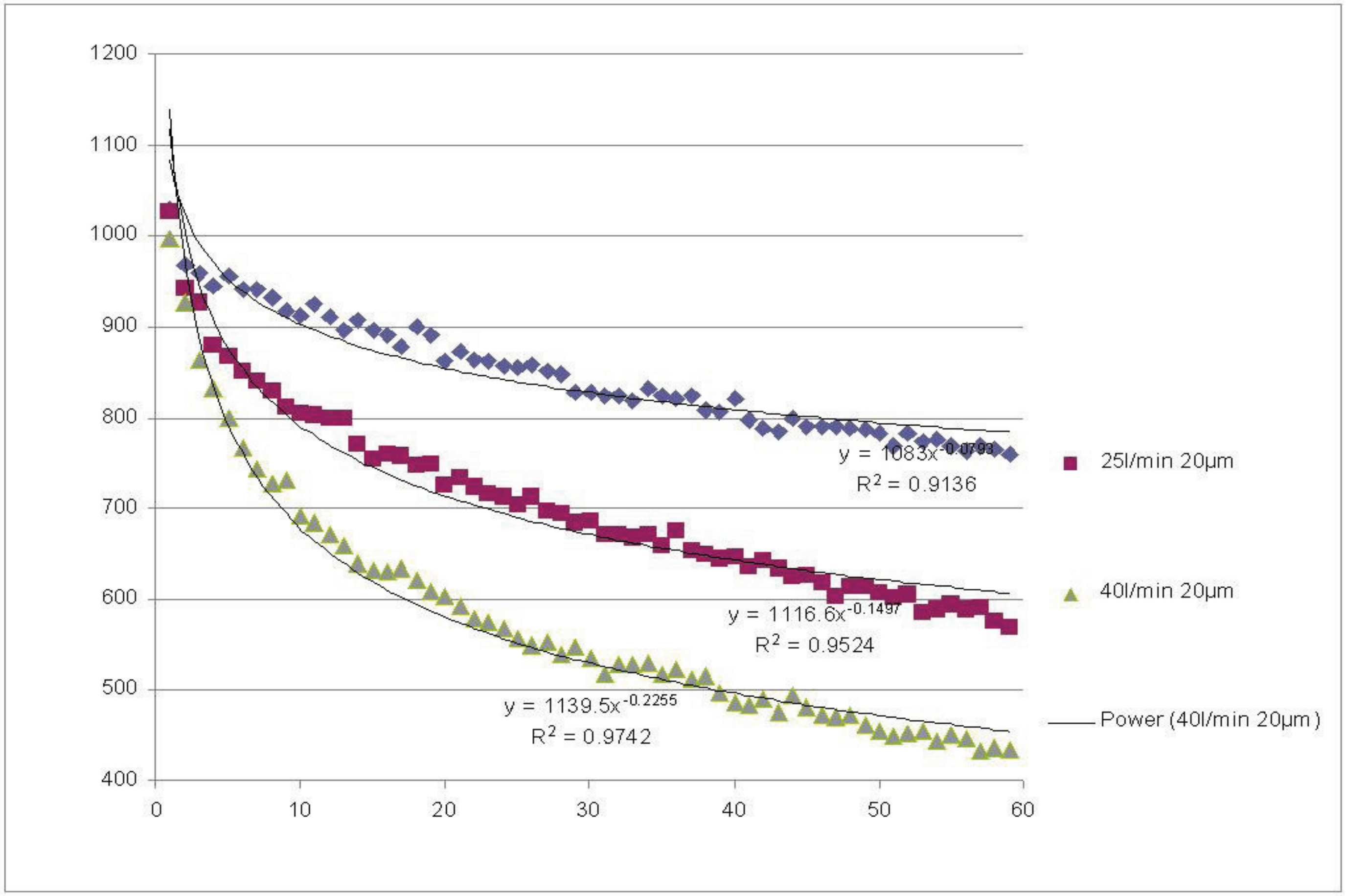


- different particle sizes
- different velocities
- different filter media
- different concentrations

Simplified to be a pre processing step for macro simulations

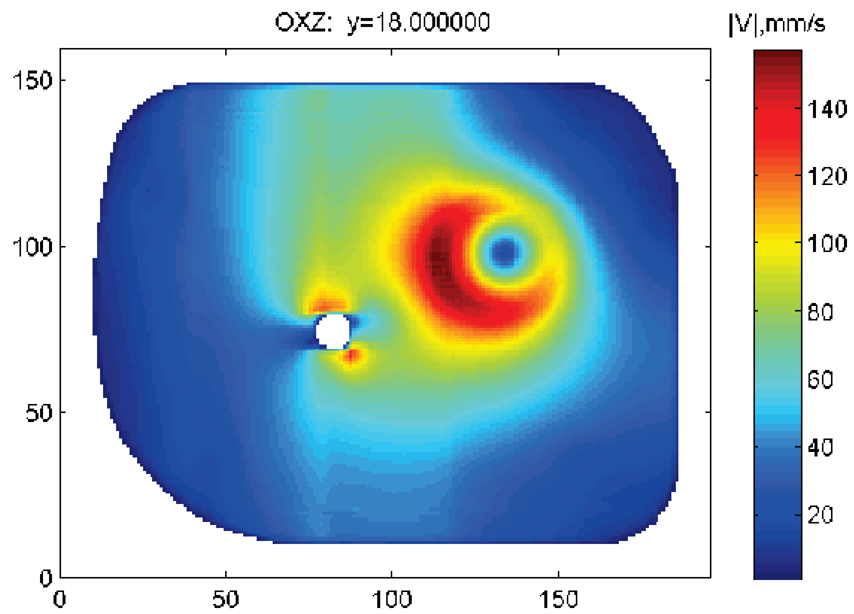
Results

Correlation tables from measurements



Results

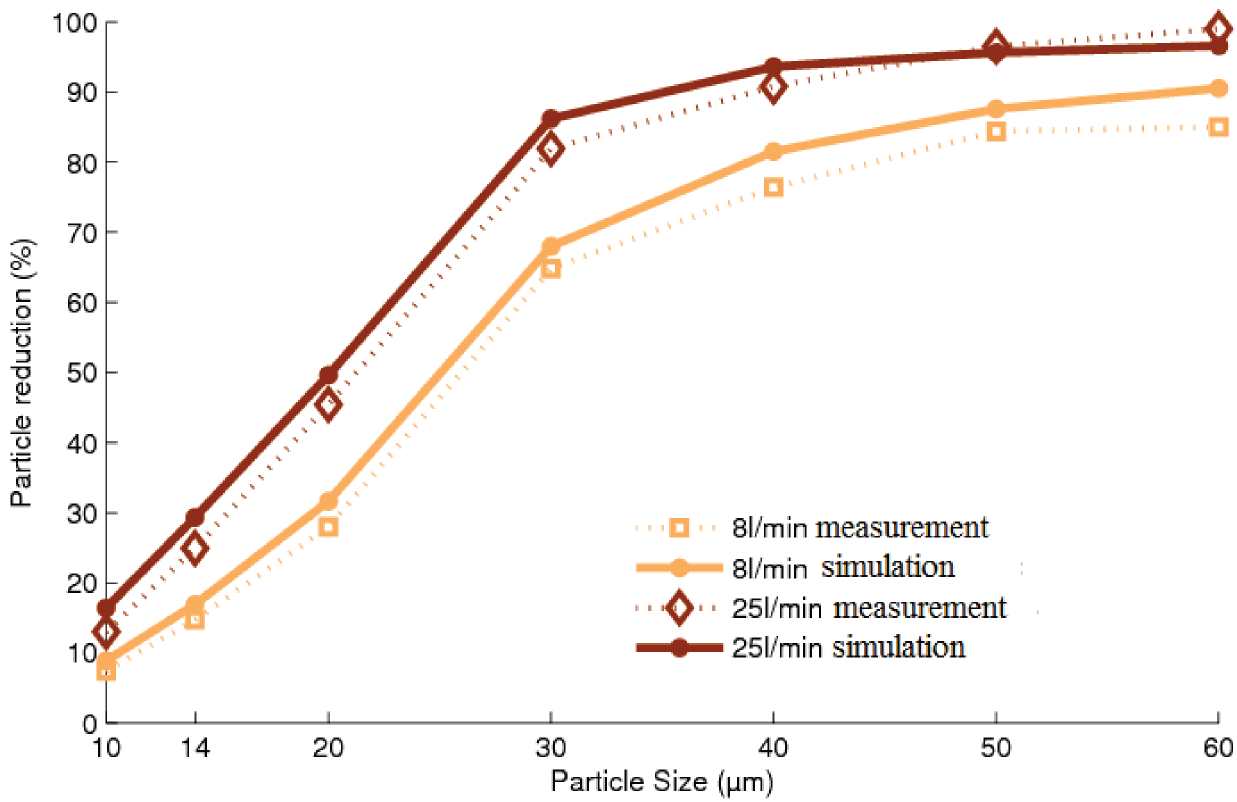
Macro simulation performed on filter element level



$$\alpha^J(y,z) = \frac{(h + \Delta t u(y,z))(n^J(t) - n^J(t+\Delta t))}{h \Delta t n^J(t)}$$

Results

Efficiency profile for different particle sizes



Conclusions

- Filtration processes vary at different scales
  - Macroscale: Navier Stokes Brinkmann system
  - Mesoscale: Particle Concentration Equation
  - Microscale: Stokes and particle deposition model
- Micro-meso-macro coupling
  - Filter element level simulation
  - Downscaling of local velocities and particle concentrations
  - Correlation tables - from microscale simulations/measurements
  - Deriving α from correlation tables

THANK YOU FOR YOUR ATTENTION!

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