

Combining Pore Morphology and Flow Simulations to Determine Two-Phase Properties of 3D Tomograms

Jürgen Becker^{1*}, Erik Glatt¹, Andreas Wiegmann¹

Fraunhofer ITWM, Fraunhofer-Platz 1, 67663 Kaiserslautern, Germany

* Corresponding author, e-mail: becker@itwm.fhg.de

Abstract — Combining Pore Morphology and Flow Simulations to Determine Two-Phase Properties of 3D Tomograms — Two-phase flow through a porous media can be described by two effective properties: the capillary pressure saturation relationship and the relative permeability. In porous media simulations on macroscopic scales, these properties are input parameters. They can be determined experimentally by measurements performed on rock samples or sand columns, and they can be determined numerically by simulations on pore scale 3D tomograms of the porous media. Numerical methods usually require large amounts of memory as tomograms consist of often more than 1000^3 grid points. Furthermore, the solution of two-phase flow problems is very time-consuming. We overcome both obstacles by combining the pore morphology method with single-phase flow simulations. The pore morphology method [1] uses the Young-Laplace equation and geometric analysis of the pore space to determine the phase distribution at a given capillary pressure. For a given capillary pressure, this phase distribution is assumed to be immobile. The relative permeability is then determined by solving Stokes' equation in the space occupied by one phase only. Using a numerically efficient solver [2], which works directly on the segmented tomogram, the computational costs of this new approach are comparatively low and results can be achieved on a desktop computer. The whole workflow: import of the tomogram; segmentation; pore morphology method; flow simulations; is integrated in the GeoDict software package [4]. This allows for an automated and rapid analysis of a given tomogram.

INTRODUCTION

The workflow to determine the two-phase properties of a porous medium numerically is presented in a case study. Input is a 3D tomogram. This is then used as a sample structure in all numerical simulations.

1 TOMOGRAM

The data set considered in this case study is a micro CT image of a Palatine Sandstone sample. The image was obtained by F. Enzmann et al at the University of Mainz, Institute for Geosciences. The image data consists of $1024 \times 1024 \times 1024$ grey-valued voxels with a resolution of $0.7 \mu\text{m}$ per voxel.

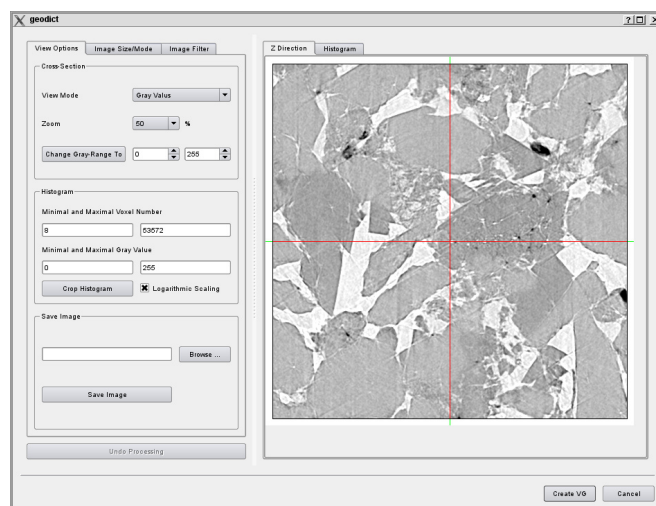


Figure 1:

A single slice of the 3D tomogram data before segmentation.

The grey-valued 3D image shown in Figure 1 is segmented into pores and solid (mineral matrix, blue) as shown in Figure 2. This is done by choosing a threshold for the grey values.

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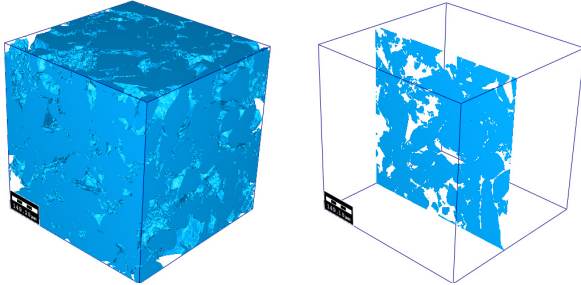


Figure 2:

3D structure after segmentation of the tomogram data.

The resulting 3D voxel mesh can be used to calculate the effective properties of the sandstone. For the sake of computational speed we down-scaled the mesh to 512 x 512 x 512 voxels with a resolution of 1.4 μm per voxel. This allowed us to perform all computations on a computer with 16 GB of RAM of which at most 11 GB were utilized. On the original size, the required amount of RAM and the run times would approximately be eightfold.

2 PORE SIZE DISTRIBUTION

The segmented sandstone sample has a porosity of 25.7%. As visible in Figures 1 and 2, the pore space consists of large pores and small connecting cracks. The geometric pore size distribution is shown in Figure 3.

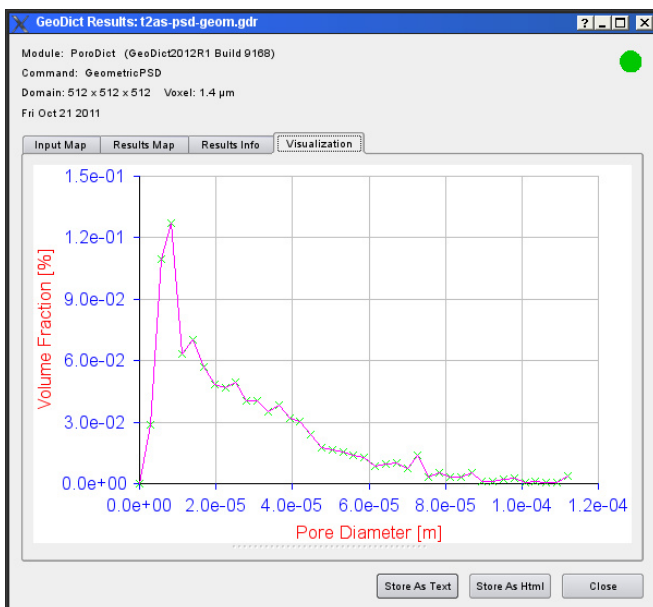


Figure 3:

Pore size distribution of the 3D tomogram shown in Figure 2.

Here, the pore radius is defined by embedding spheres into the pore volume: a voxel belongs to a pore of radius r if r is the radius of the largest sphere, embedded in the pore space, containing this voxel.

3 CAPILLARY PRESSURE

The Young-Laplace equation gives a relationship between the pore radius r and the capillary pressure p_c by:

$$(1) \quad p_c = \frac{2\gamma \cos \vartheta}{r}.$$

Here γ denotes the surface tension and ϑ denotes the contact angle. This allows calculating the capillary pressure curve from the pore size distribution. But this direct calculation would not take the connectivity of the pores into account. This is overcome by the *pore morphology method* [1, 3]. This method uses (1) but also guarantees the connectivity of the intruding phase to a reservoir. A *First Drainage* experiment is simulated by assuming i) initially water-filled pores, ii) an air reservoir on top of the sample, and iii) a water reservoir at the bottom of the sample. When air is pushed in, some water-filled pores may get disconnected from the water reservoir. If this happens, the water is considered as trapped and will stay in the medium as residual water. Fig. 4 and 5 show the results of this method, assuming a contact angle of 0° . Numerically, we found a residual water content of 8.6% of the pore space for this sandstone sample.

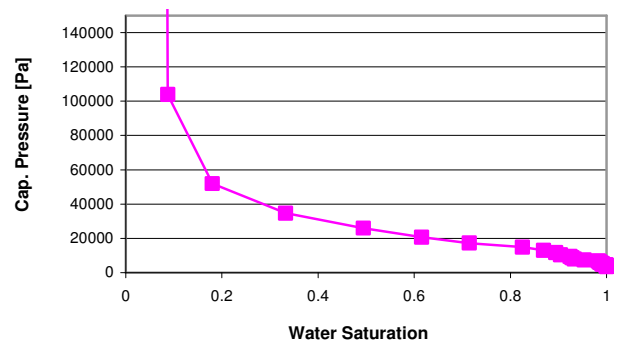


Figure 4:

Capillary pressure curve shown for a First Drainage simulation.

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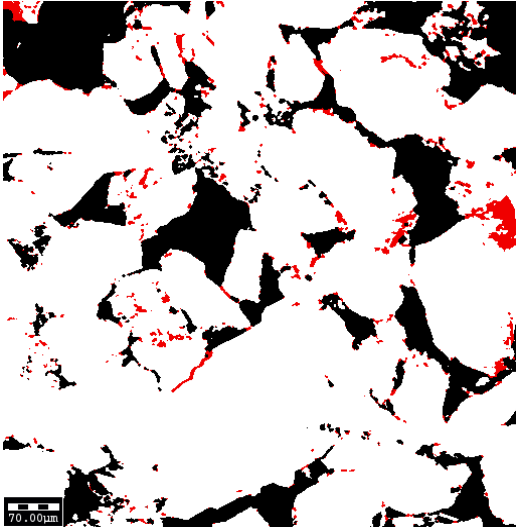


Figure 5:

Residual water distribution is shown after a First Drainage simulation. The image shows a single slice of the 3D structure. Air-filled pores are black, water-filled pores are red and the solid matrix is white.

4 PERMEABILITY

The permeability κ of the sandstone is defined macroscopically by Darcy's law

$$(2) \quad \bar{u} = -\frac{\kappa}{\mu} \nabla p.$$

Here μ denotes viscosity, \bar{u} is the average velocity and ∇p is the pressure drop over the media. κ is found by determining the flow field on the pore scale, which is governed by Stokes' equation

$$(3) \quad -\mu \Delta u + \nabla p = 0.$$

We solve (3) using an FFT-based finite volume solver [2], which is optimized for large voxel grids, i.e. no re-meshing is necessary. The computational costs for 512 x 512 x 512 grid points are as follows: we used 4 processes on a desktop machine and needed 2.75 GB RAM per process (11 GB RAM in total) and 8.5 h until convergence.

For flow in z-direction, the calculated permeability of the sandstone sample was approximately 1 darcy, precisely: $0.966 \times 10^{-12} \text{ m}^2$.

5 RELATIVE PERMEABILITY

If the media is partially saturated, the permeability κ is not constant, but depends on the water saturation s . We determine $\kappa(s)$ by a combination of the methods described in Sections 3 and 4. Here, we use the fact that the pore morphology method provides both a capillary pressure curve and also a distribution of air and water phase for each saturation level. Assuming that for a given saturation this distribution is static in the case of slow Darcy/Stokes flow, $\kappa(s)$ is found by solving a single-phase flow problem as described in Section 4 in the water phase only. Figure 6 illustrates this idea: some pores are blocked with air bubbles and the flow only uses the water-filled pores.

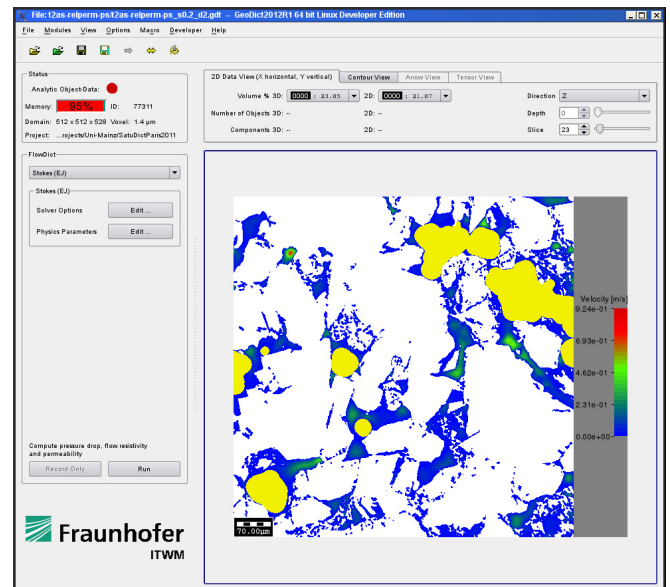


Figure 6

Visualization of a slice of the flow field with GeoDict [4]. The sandstone matrix is shown in white, yellow areas are air-filled, blue to red indicates the velocity in the water phase.

Figure 7 shows the resulting $\kappa(s)$ curve. In this simulation, water and air were distributed due to Eq. (1).

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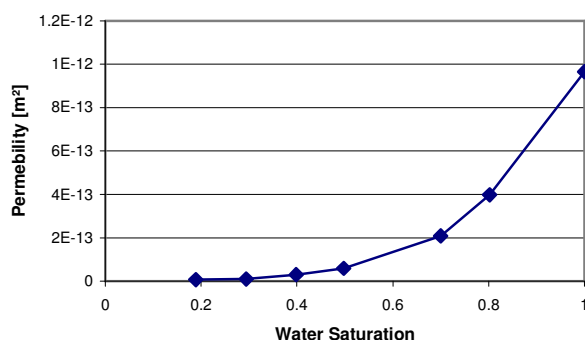


Figure 7:

Saturation dependent water permeability, flow in z-direction.

6 CONCLUSIONS

In this paper, we have shown how to determine the relative permeability of a rock from a 3D tomogram of a rock sample. All steps have been performed with the GeoDict software on a desktop computer.

Additionally to the shown results, some variations of the workflow are possible:

- Instead of drainage, imbibition can be considered when calculating the capillary pressure curve. Also, both wetting models can be used when calculating the relative permeability.
- Instead of water permeability, air permeability can be calculated by considering the flow in the air-filled pores only.
- Similar to the approach taken to calculate (relative) permeability, (relative) diffusivity can be determined by solving Laplace's equation.

The approach is not limited to rock samples, but can be used for all porous media. Examples are gas diffusion layers of fuel cells [5,6], and fibrous nonwoven sheets [7,8]

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