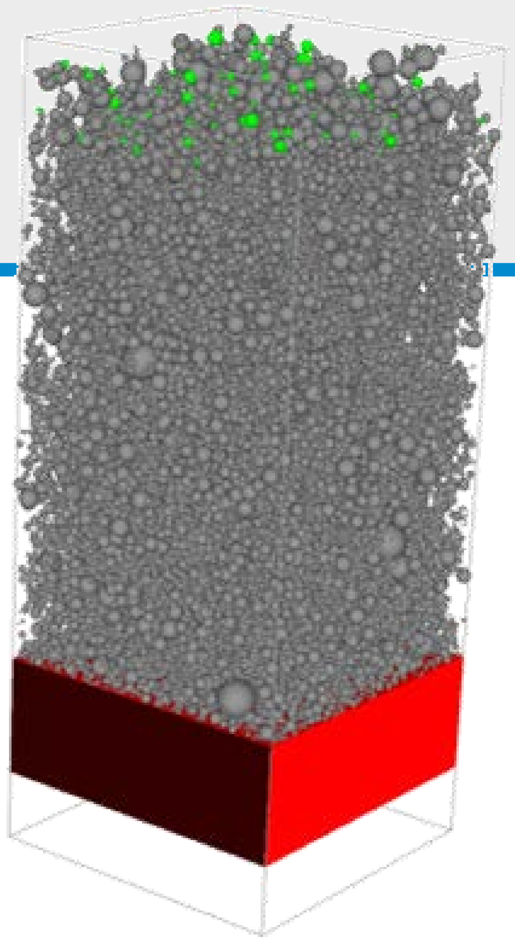
Simulating Filter Cake Formation At Different Resolutions

AFS 2015

April 28th, 2015

Charlotte



Cornelia Kronsbein
Jürgen Becker
Liping Cheng
Andreas Wiegmann

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Who is Math2Market?

- Math2Market GmbH was founded September 2011 in Kaiserslautern.
- Spin-off of Fraunhofer Institute for Industrial Mathematics, ITWM.

- GeoDict Software
  - Sales
  - Development and Customization
  - Consulting

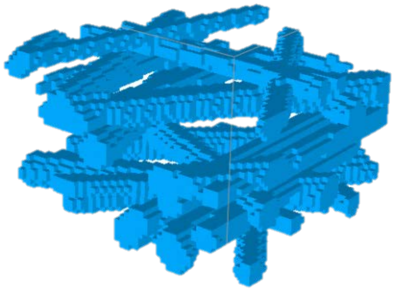
- Table top 11

Simulating Filter Cake Formation At Different Resolutions

Overview

1. State of the art: cake filtration at different resolutions.
2. What is the problem with polydisperse particles?
3. How can we solve it?

How Cake Filtration Is Simulated



1. Filter Model

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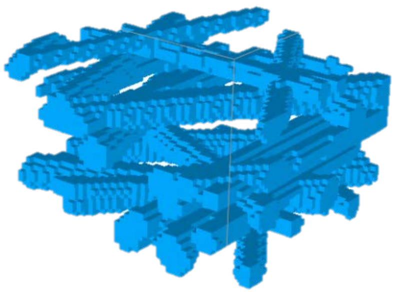

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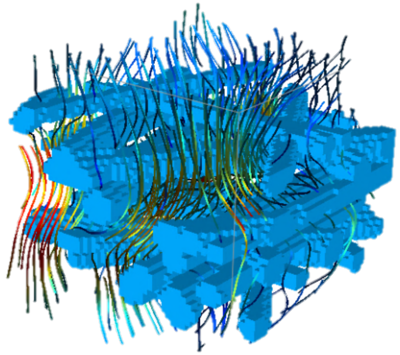

2. Flow Field

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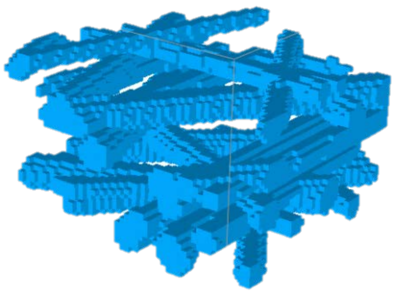

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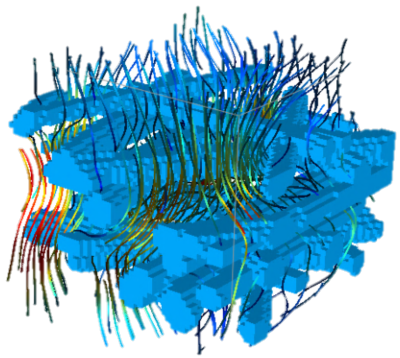

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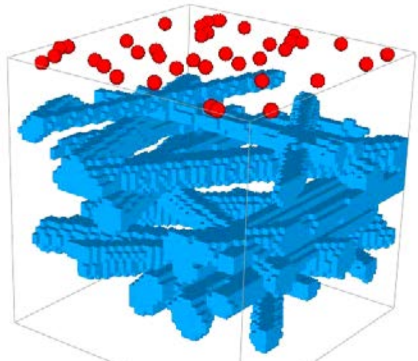
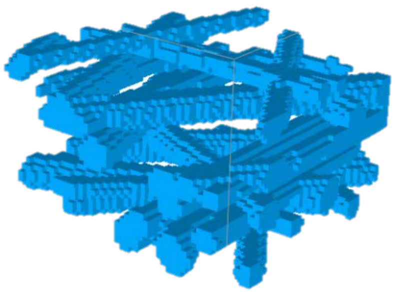


3. Track Particles

How Cake Filtration Is Simulated



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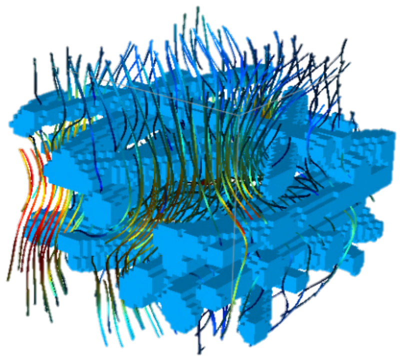

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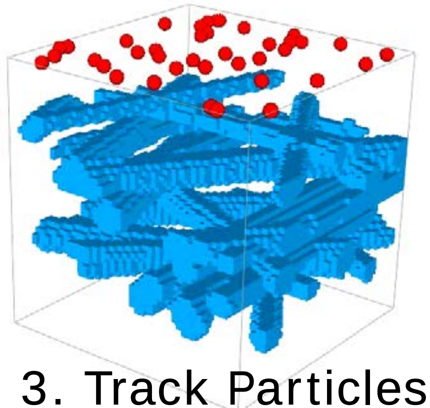

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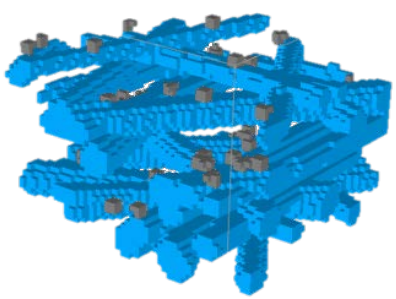
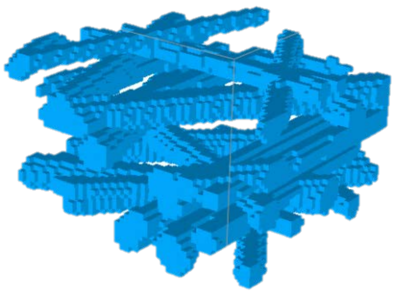


4. Deposit Particles

How Cake Filtration Is Simulated

FilterDict



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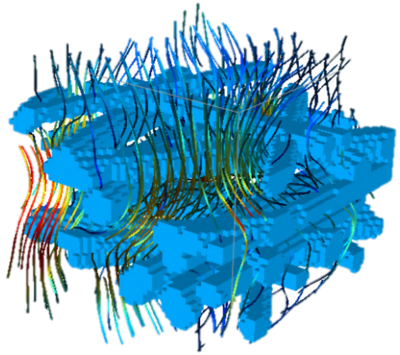

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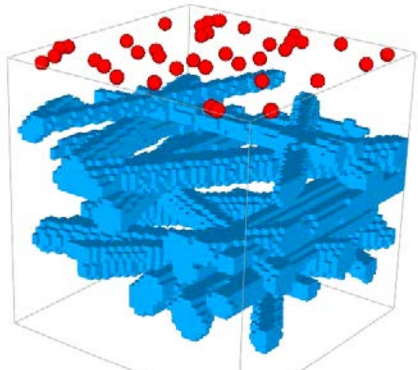

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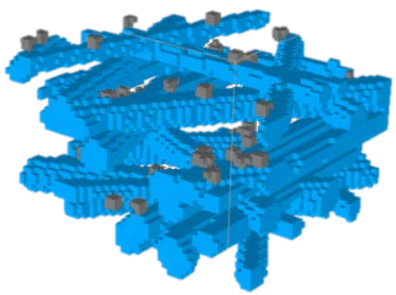

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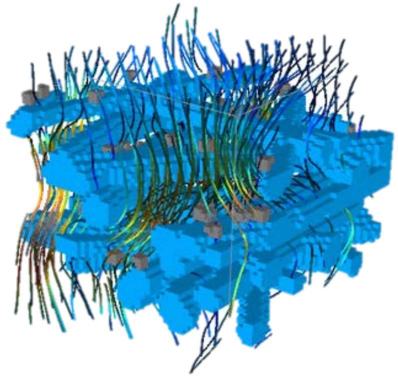

5. Flow Field

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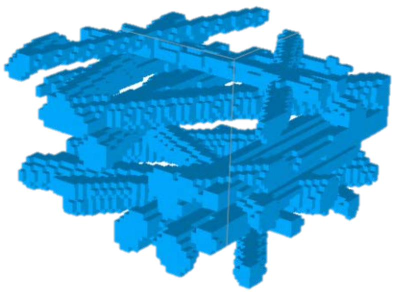
How Cake Filtration Is Simulated

FilterDict



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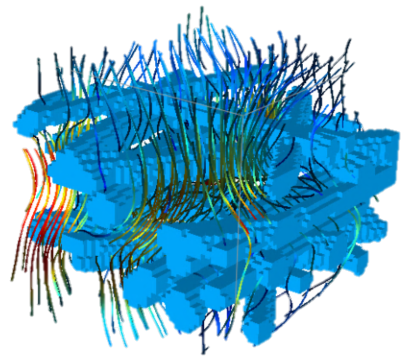

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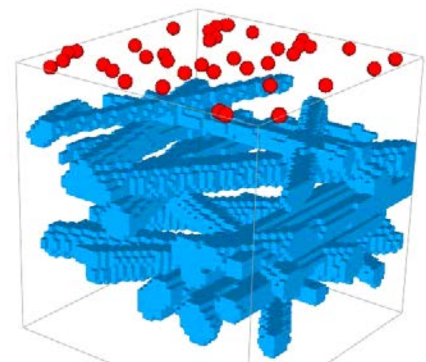

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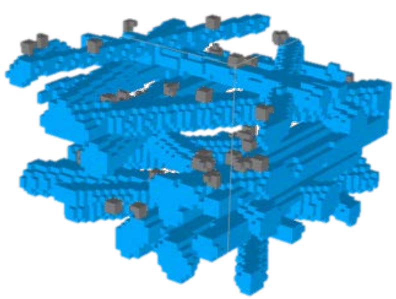

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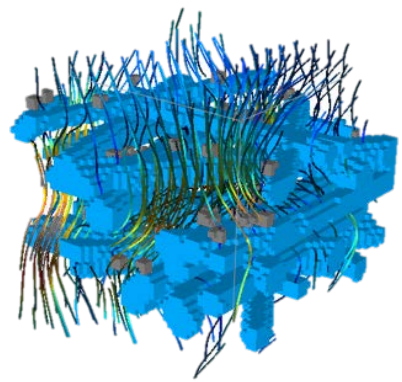

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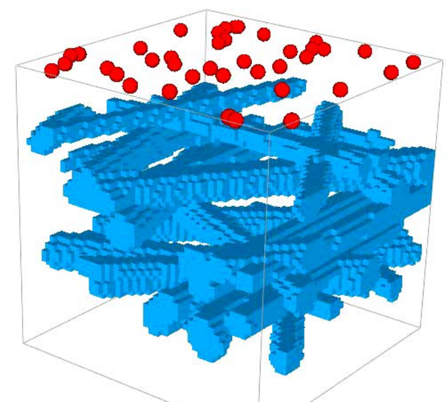

6. Repeat …

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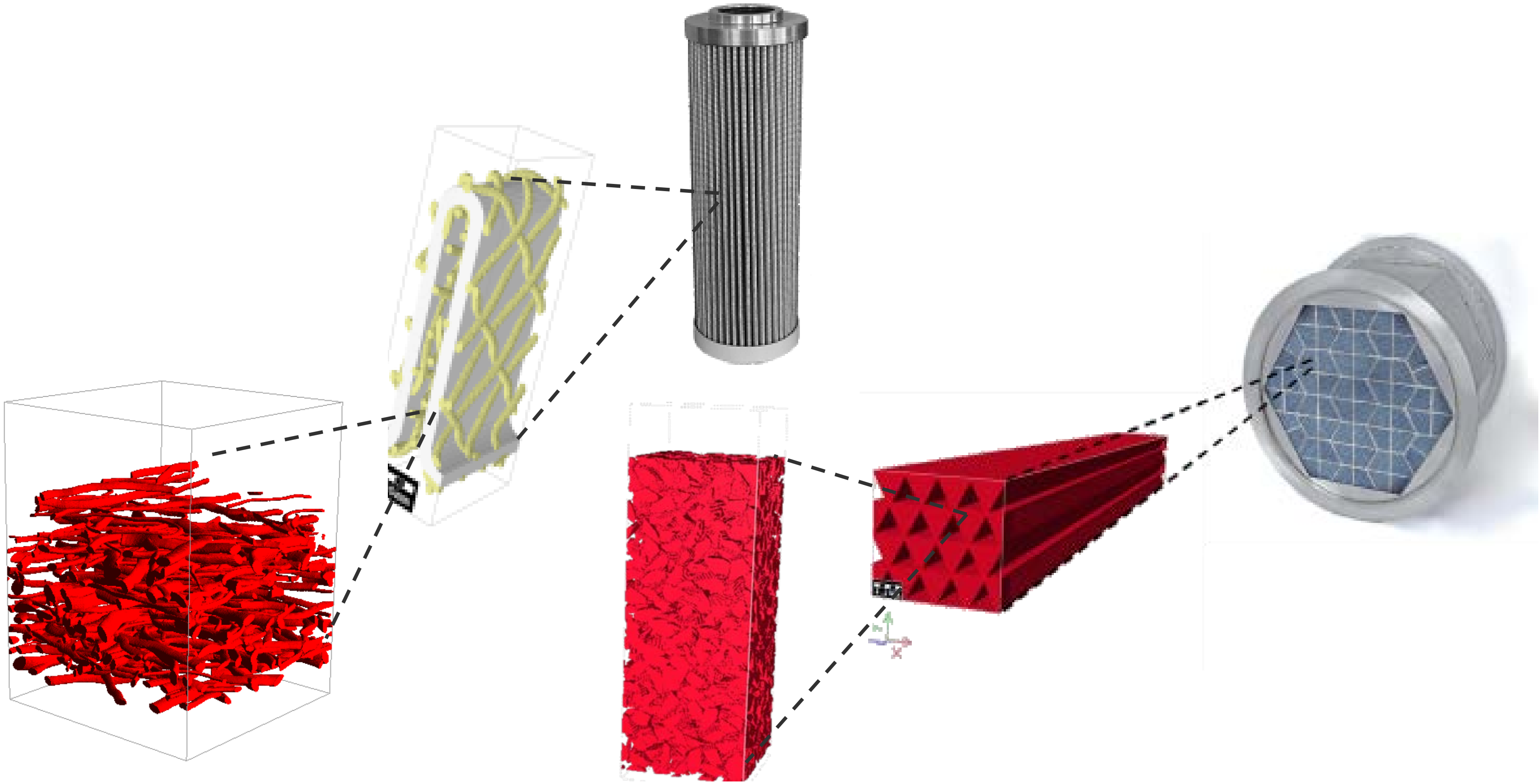
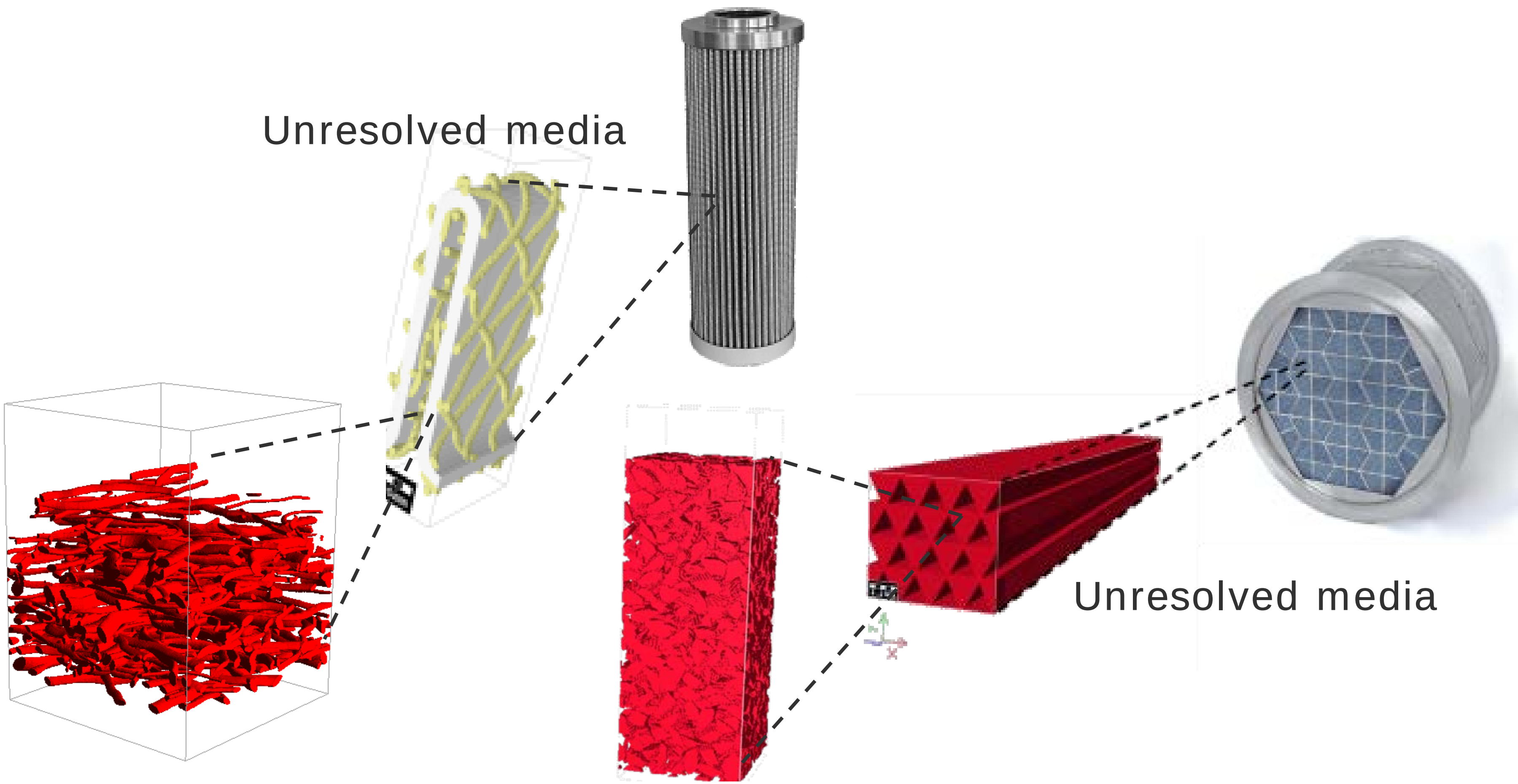
Computational Grid: Filter Media



Computational Grid: Filter Media



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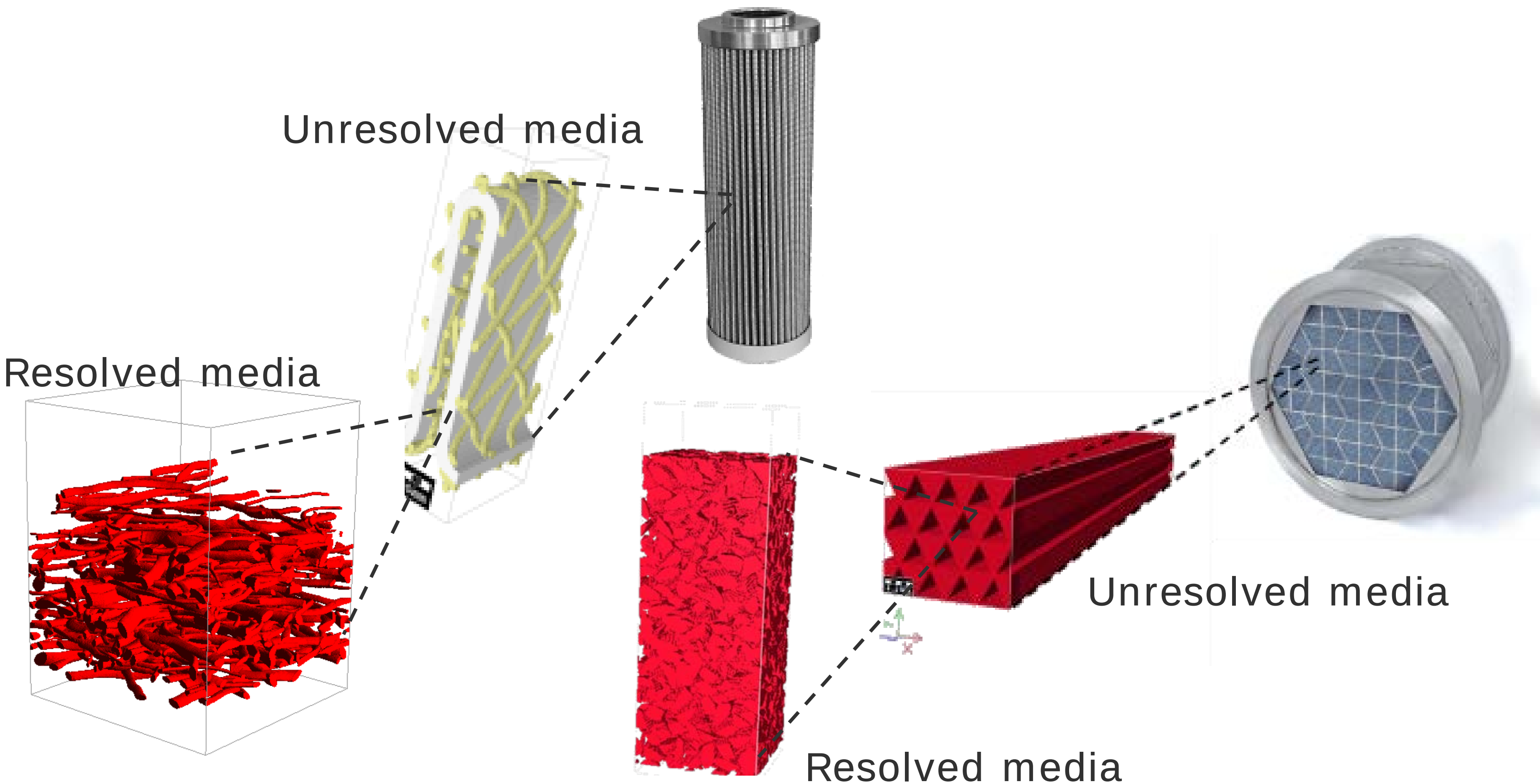
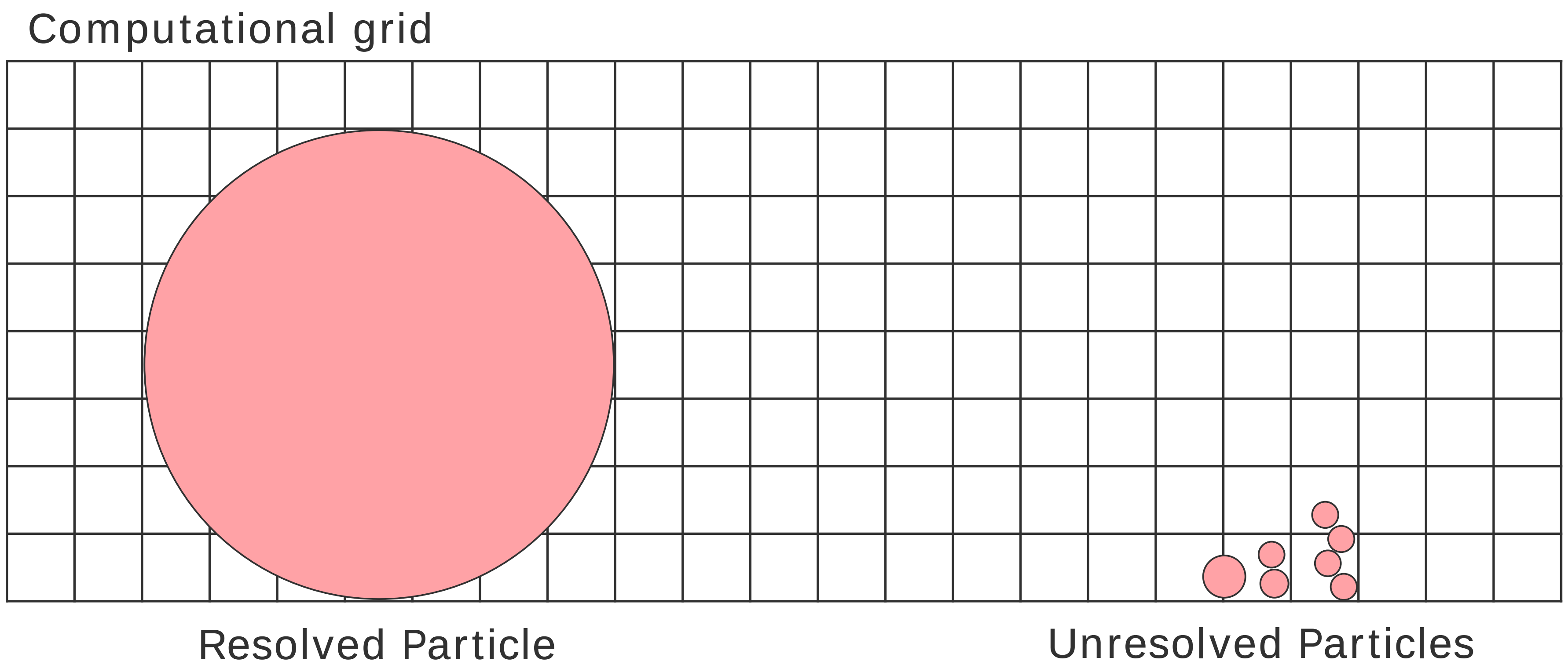
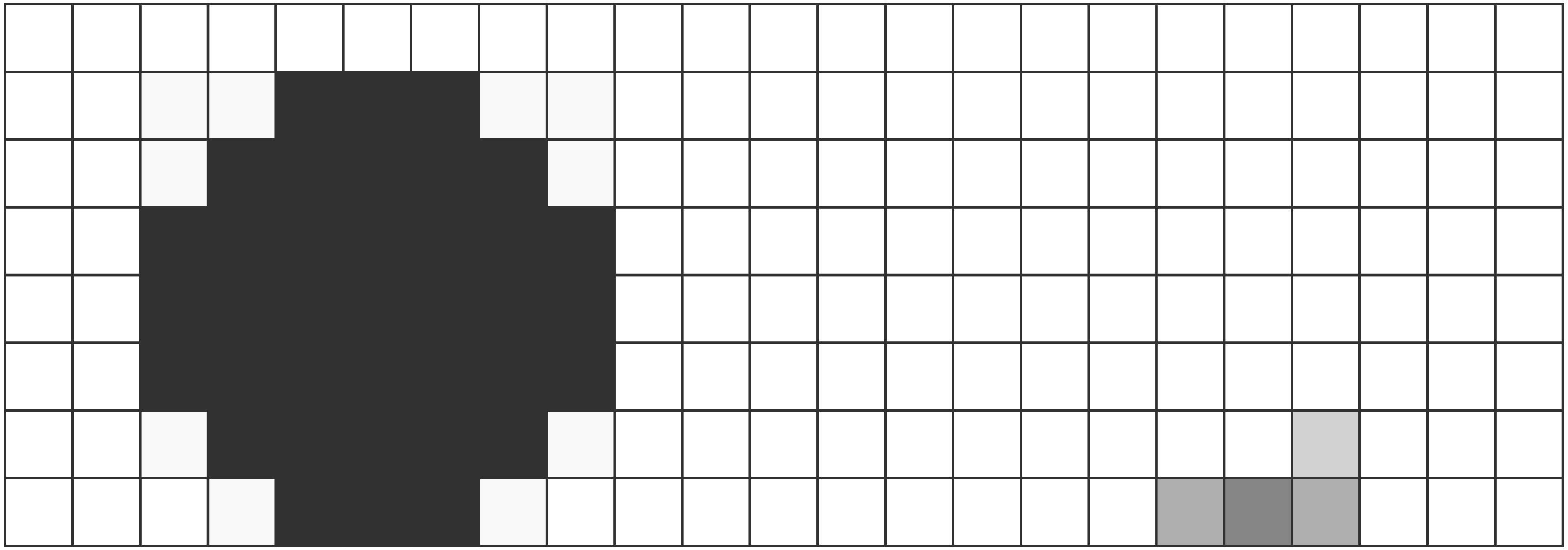


Computational Grid: Dust Particles



Computational Grid: Dust Particles

Computational grid



Resolved Particle

Empty/Solid Cells

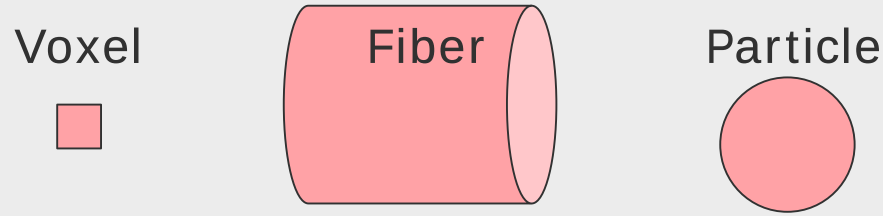
Unresolved Particles

Porous Cells

Computational Grid – 4 Cases

1. Resolved media – resolved particles
2. Resolved media – unresolved particles
3. Unresolved media – unresolved particles
4. Unresolved media – resolved particles

Resolved Media – Resolved Particles



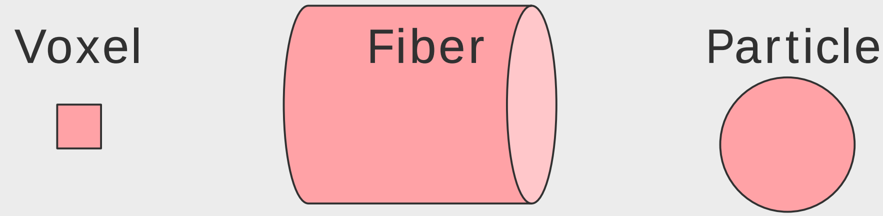
- Voxels are solid or empty

- Navier-Stokes equation

$$-\mu\Delta\vec{u} + \rho(\vec{u}\cdot\nabla)\vec{u} + \nabla p = 0, \qquad \nabla\cdot\vec{u} = 0$$

- Particles are discretized into solid/empty grid cells

Resolved Media – Resolved Particles



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Resolved Media – Resolved Particles

Voxel Fiber Particle

Example: Oil filtration – Multi Pass test

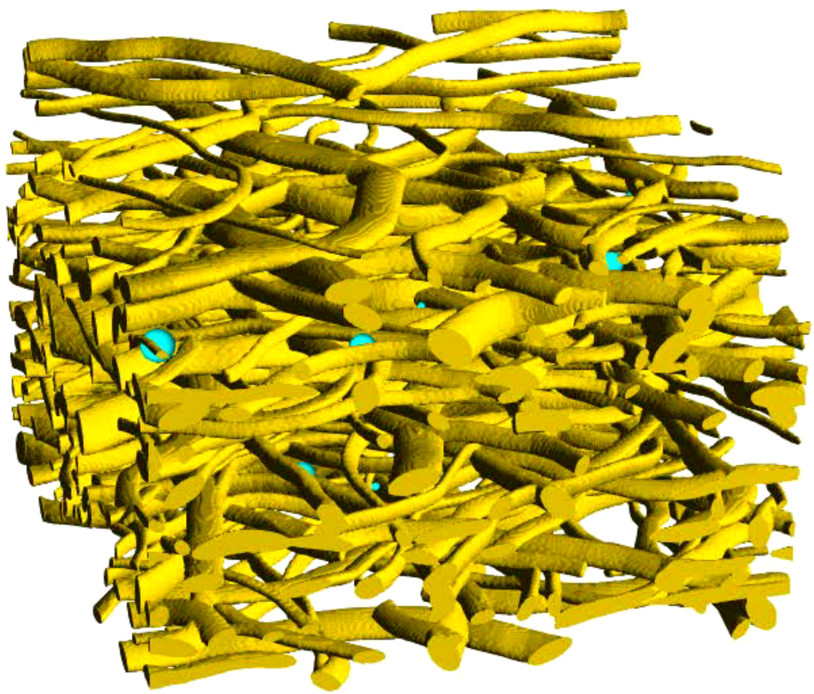
Resolved Media – Resolved Particles

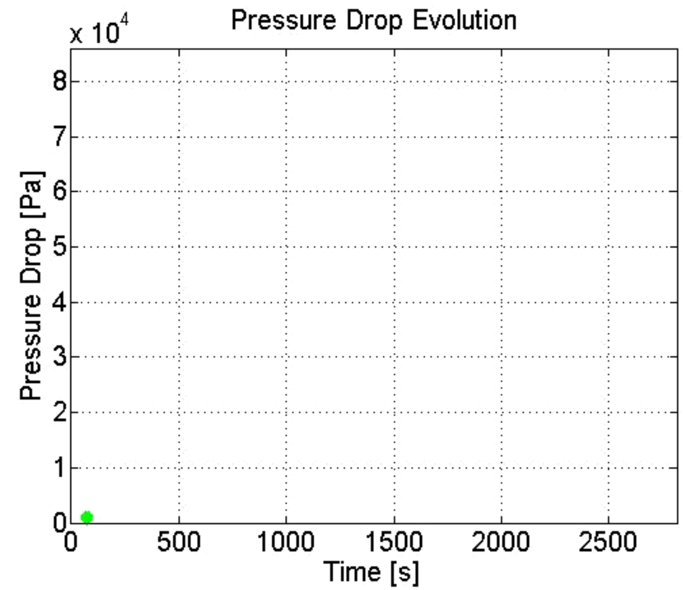
Voxel

Fiber

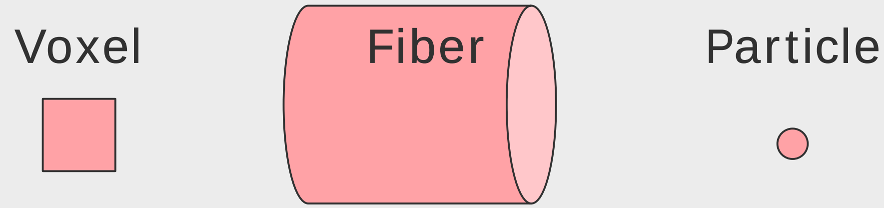
Particle

Example: Oil filtration – Multi Pass test





Resolved Media – Unresolved Particles



- Voxels are solid, empty or porous

- Navier-Stokes-Brinkman equation

$$-\mu\Delta\vec{u} + \rho(\vec{u}\cdot\nabla)\vec{u} + \frac{\mu}{\kappa}\vec{u} + \nabla p = 0, \qquad \nabla\cdot\vec{u} = 0$$

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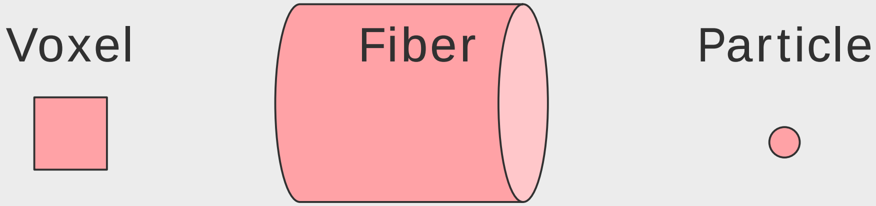

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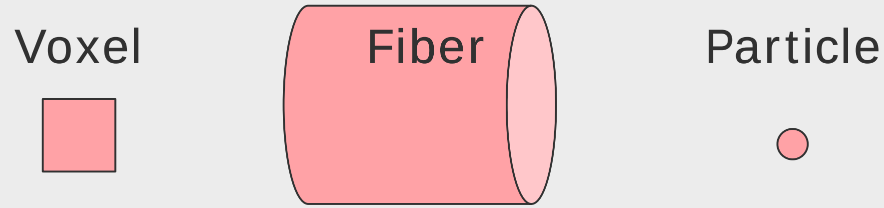

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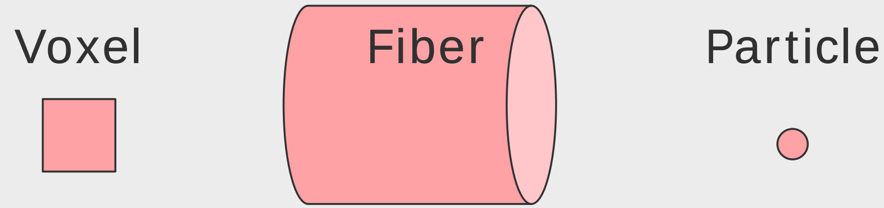
flow resistivity: $\sigma\sigma = \mu\ \kappa\ \mu\mu\ \mu\ \kappa\ \kappa\kappa\ \mu\ \kappa = \quad f \quad f\ max \quad \sigma\ max$ for $0 < f < f\ max$
is reached, no more particles can be added.

In porous voxels:

- Local solidity f changes when a particle is added.
- Local $f\ max \quad max\ x\ max$

Resolved Media – Unresolved Particles



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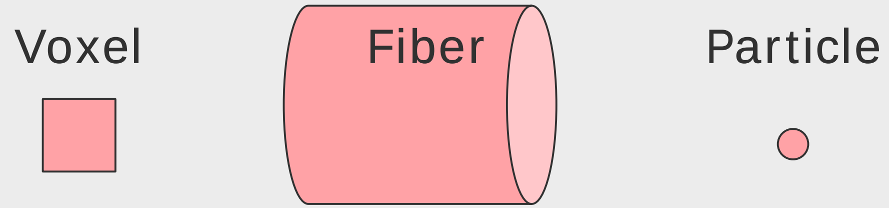
In porous voxels:

- Local solidity f changes when a particle is added.
- Local $f\ max$ $max\ x\ max$

Input parameters: $f\ max$ and $\sigma\ max$

$max\ x\ max\ x\ max$

Resolved Media – Unresolved Particles



Example: Soot filtration – ceramic filter

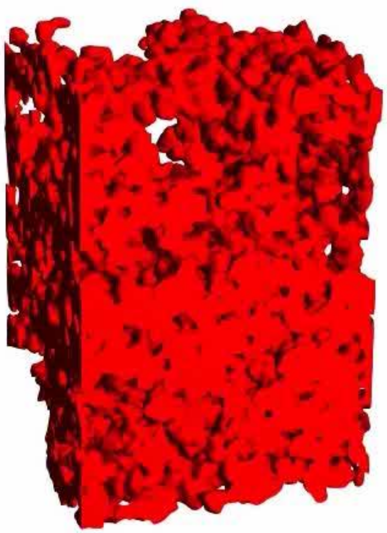
Resolved Media – Unresolved Particles

Voxel

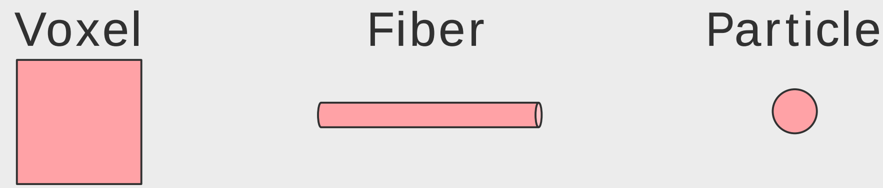
Fiber

Particle

Example: Soot filtration – ceramic filter

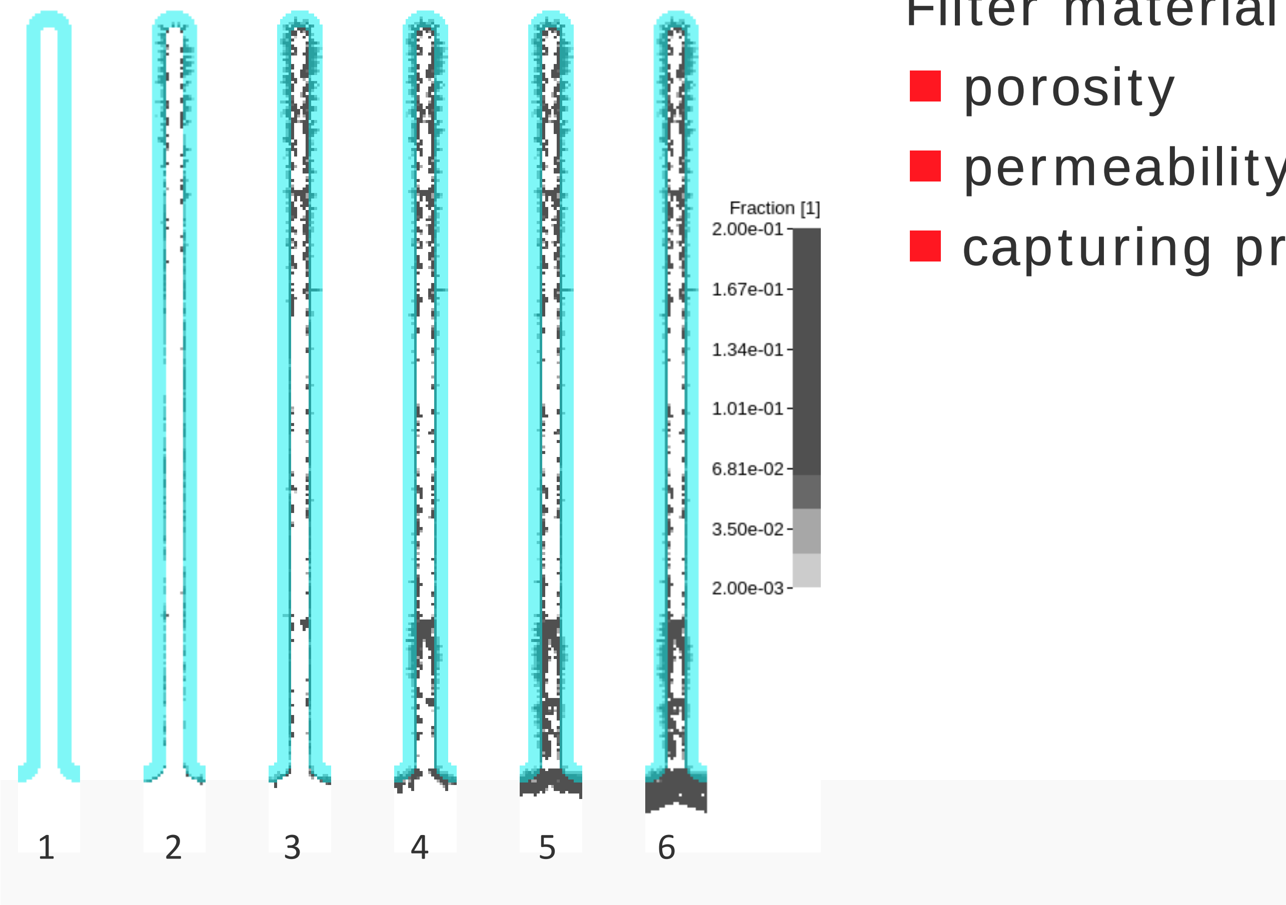


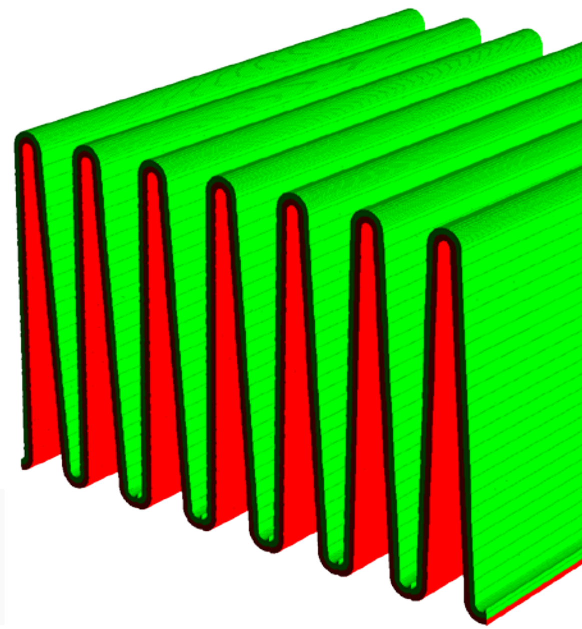
Unresolved Media – Unresolved Particles



Filter material described by

- porosity
- permeability
- capturing probability model





Everything OK ?

1. Resolved media – resolved particles
2. Resolved media – unresolved particles
3. Unresolved media – unresolved particles
4. *Unresolved media – resolved particles*

Everything OK ?

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2. Resolved media – unresolved particles
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No!
What happens if some particles are resolved and some are not?

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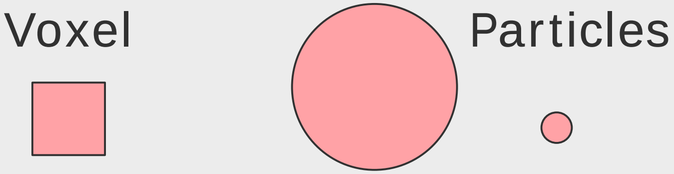
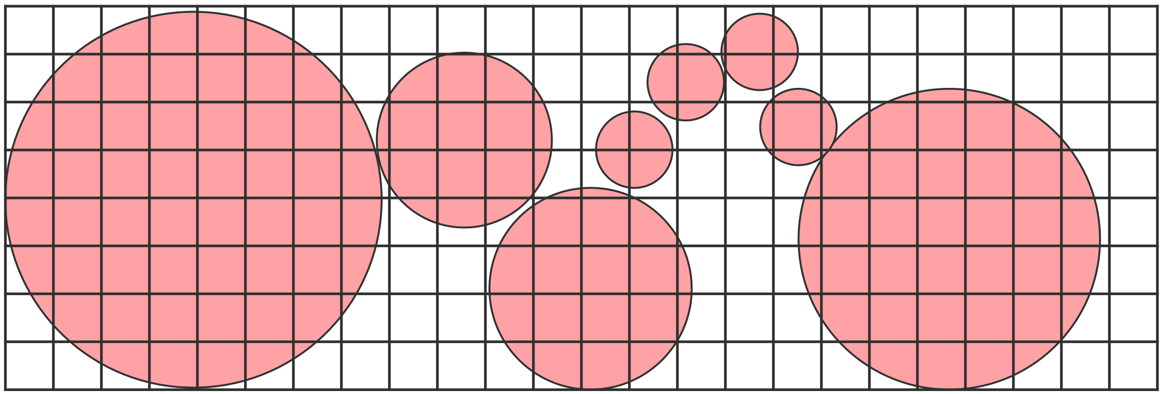
The Problem
Polydisperse Particles





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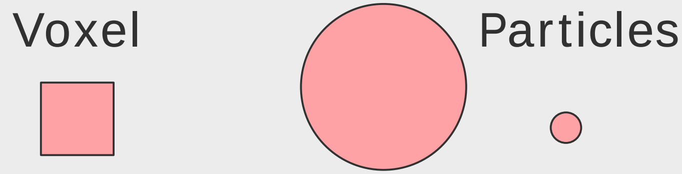
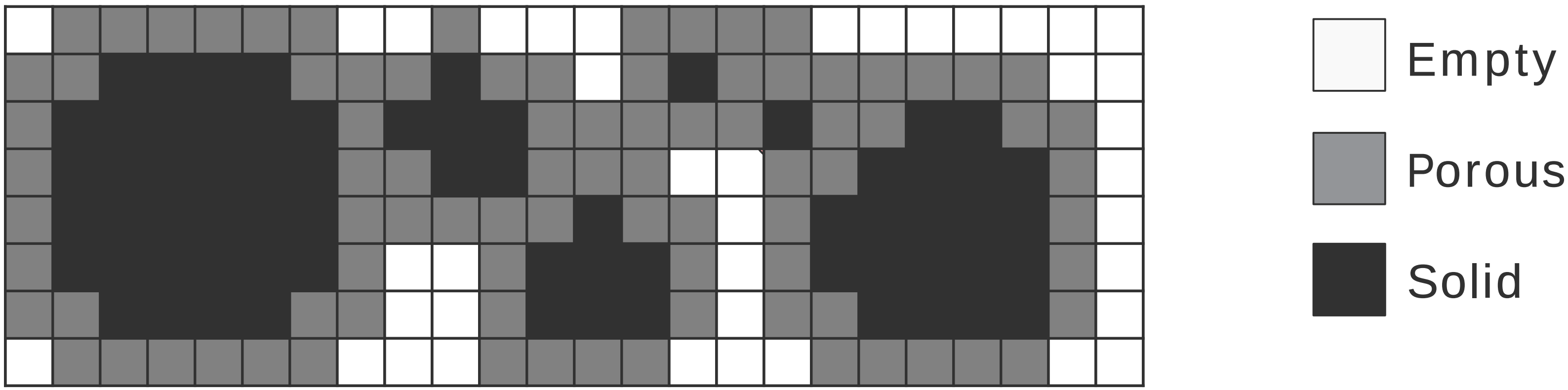
The Problem
Polydisperse Particles





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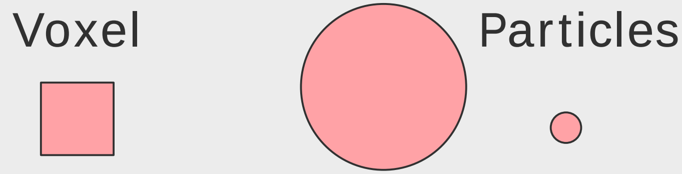
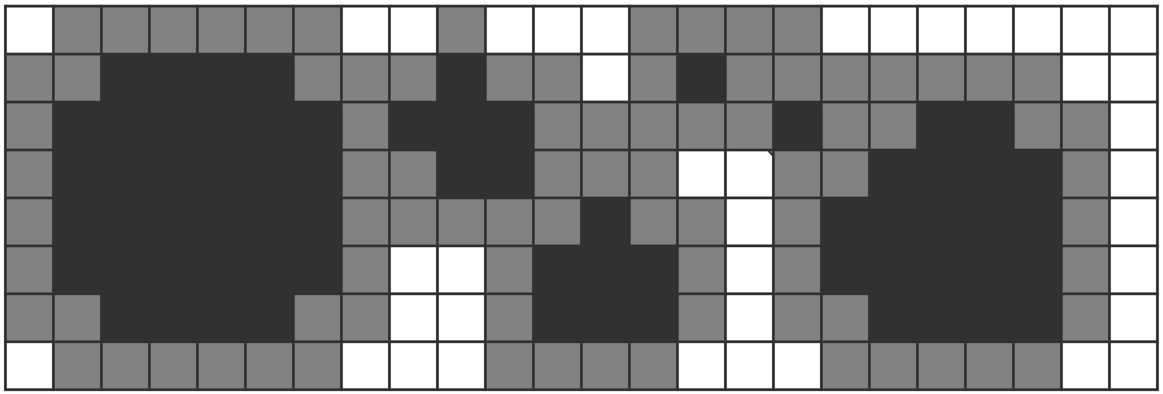
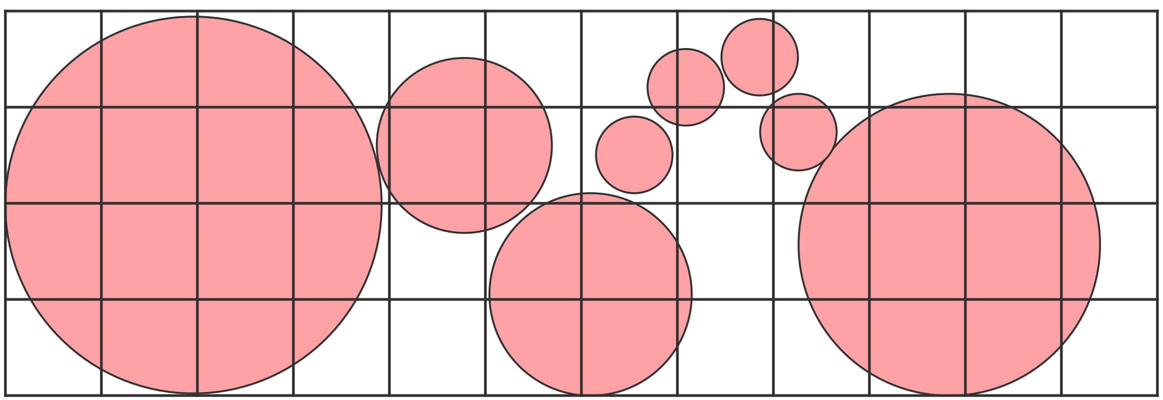




Empty

Porous

Solid



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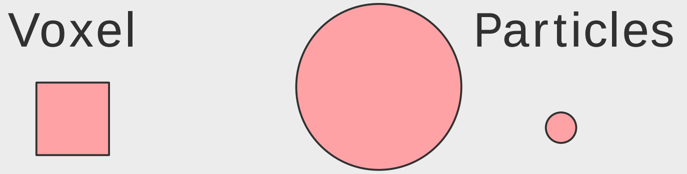
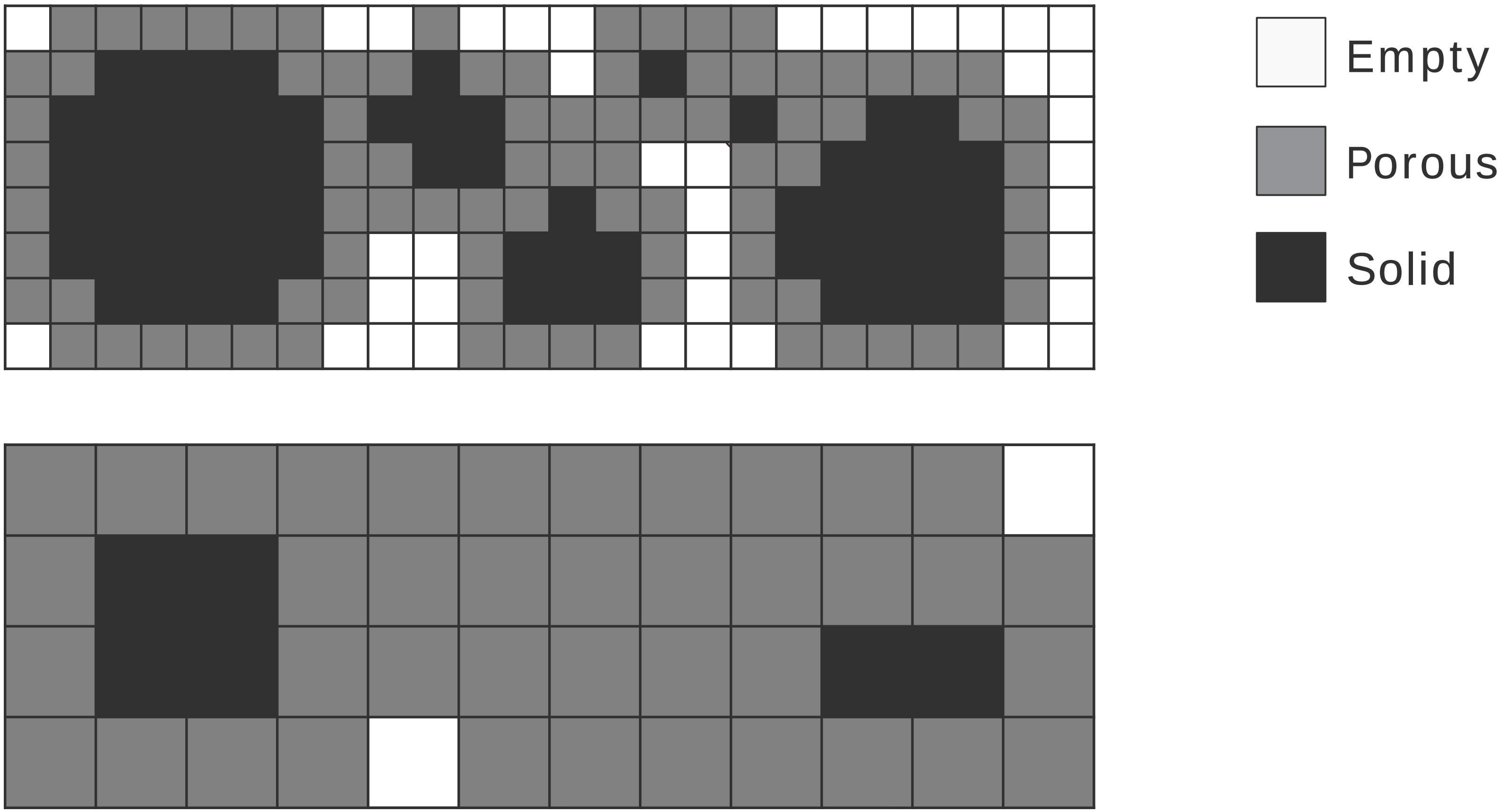
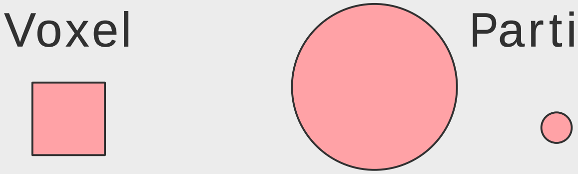


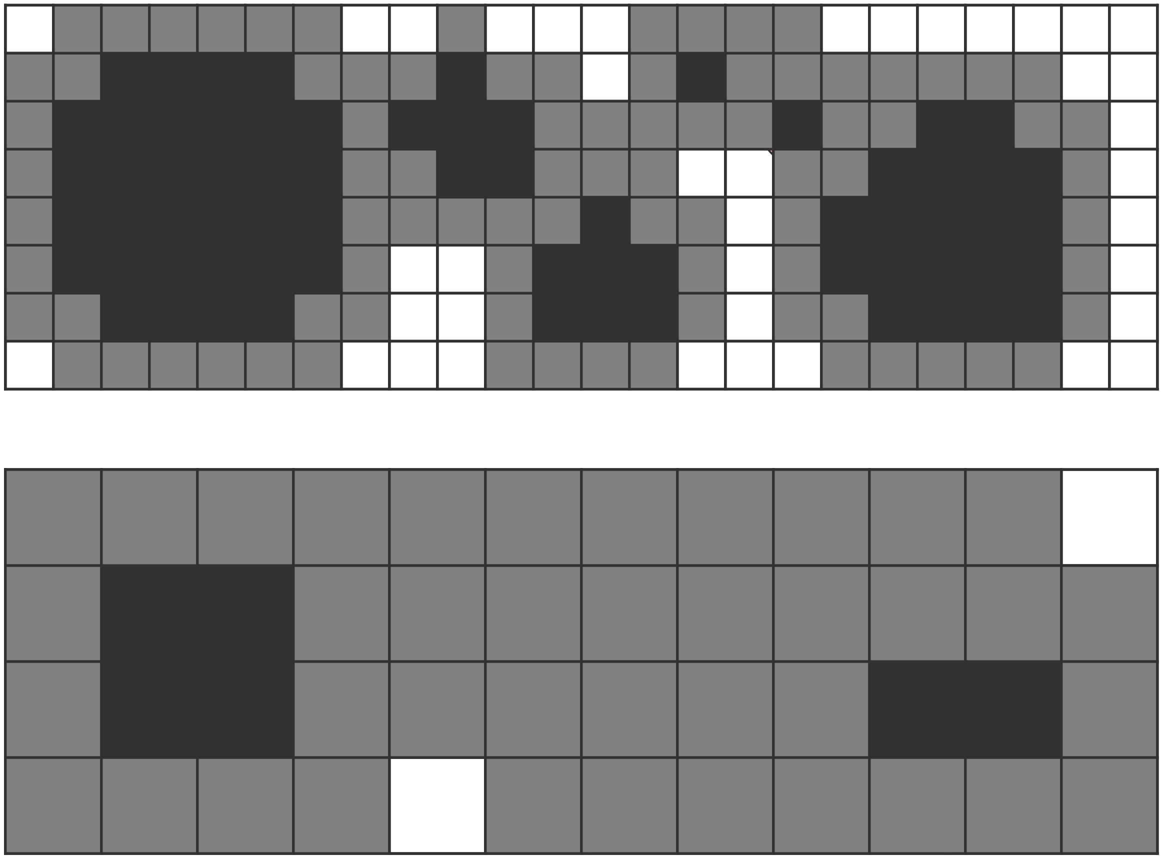


The Problem
Polydisperse Particles

Voxel

Particles





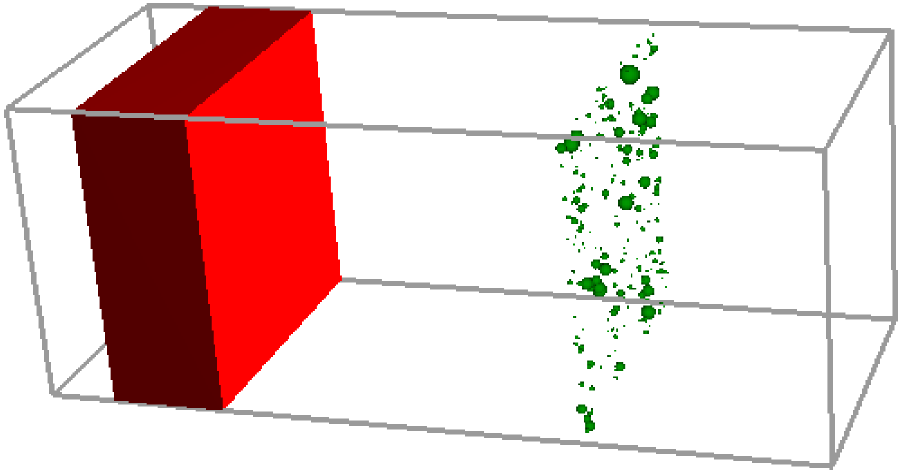
Empty

Porous

Solid

Simulation result should not depend on resolution!

Simulation Setup



Air flow with particles

- No depth filtration, only cake filtration
- Unresolved filter media: 48 µm thick, fixed permeability
- Step 1: Particles are *caught on first touch*
- Step 2: Particles are *sieved*

1. Vary resolution between 0.5 µm per voxel and 24 µm per voxel
2. Determine flow resistivity and cake solidity

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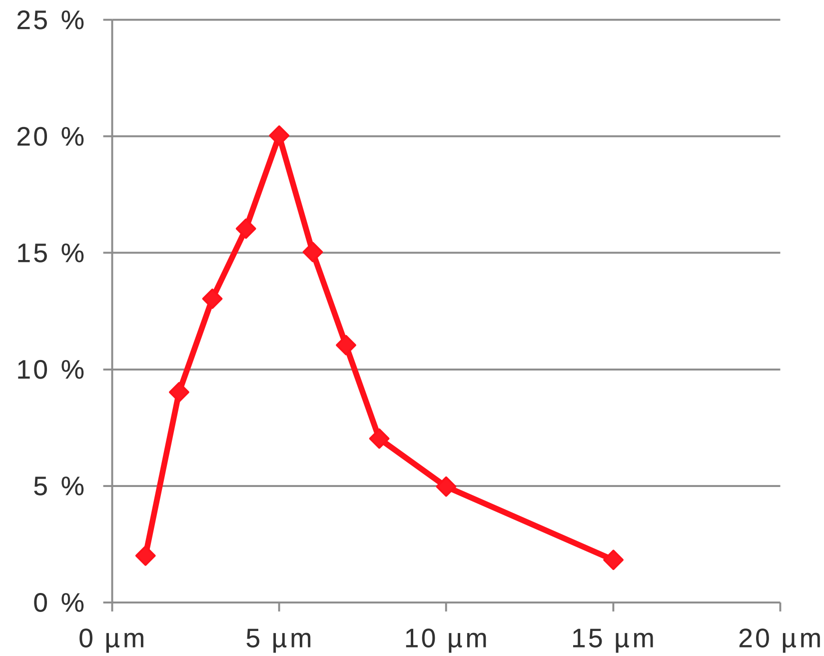
Particle Size Distribution

Particle Diameter	Mass %	Count %
1 µm	2.00	48.340
2 µm	9.01	27.190
3 µm	13.03	11.640
4 µm	16.04	6.040
5 µm	20.04	3.870
6 µm	15.03	1.680
7 µm	11.03	0.780
8 µm	7.03	0.330
10 µm	4.97	0.120
15 µm	1.82	0.013

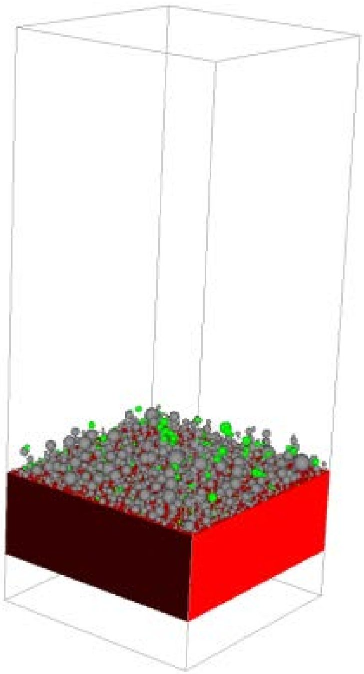


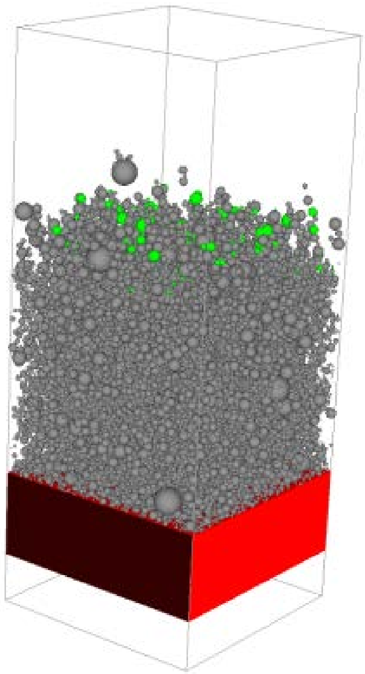
SAE Ultrafine Dust (ISO 12103-1)

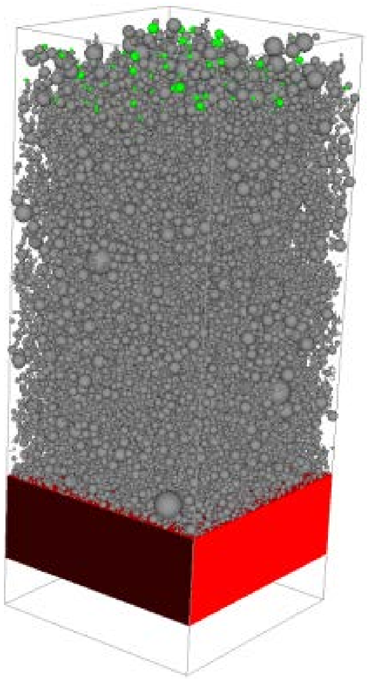
Step 1: Caught On First Touch

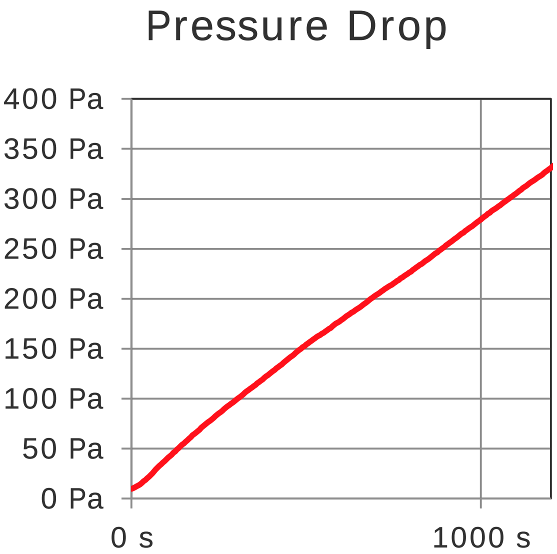
Fully Resolved Particles
Step 1: Caught On First Touch

Resolution 0.5 µm



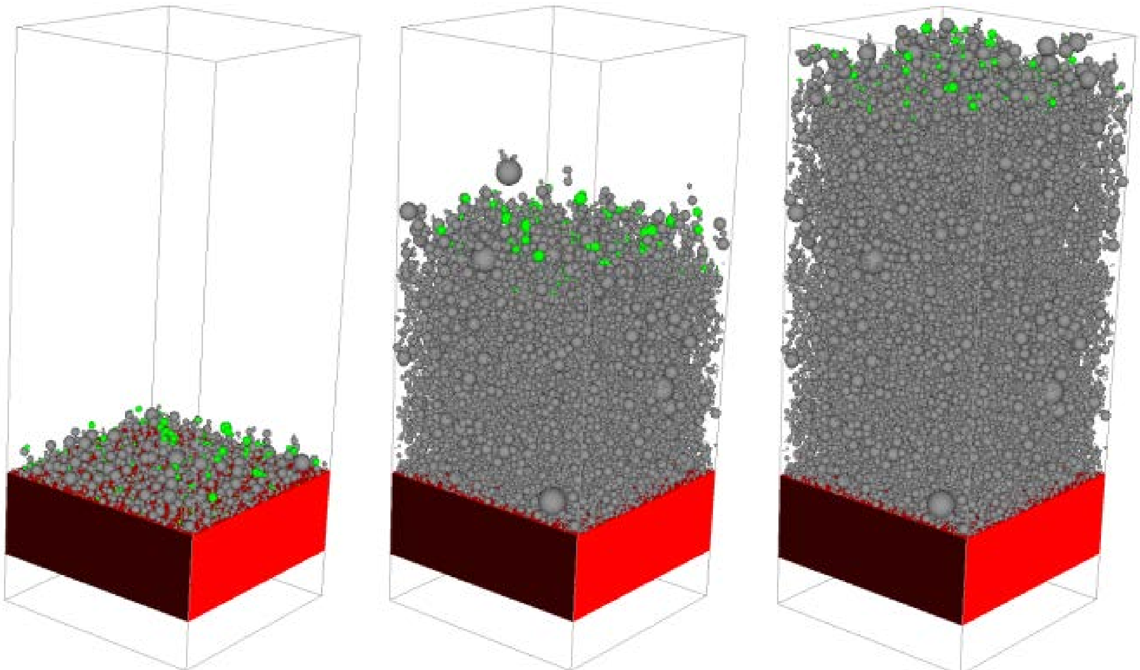


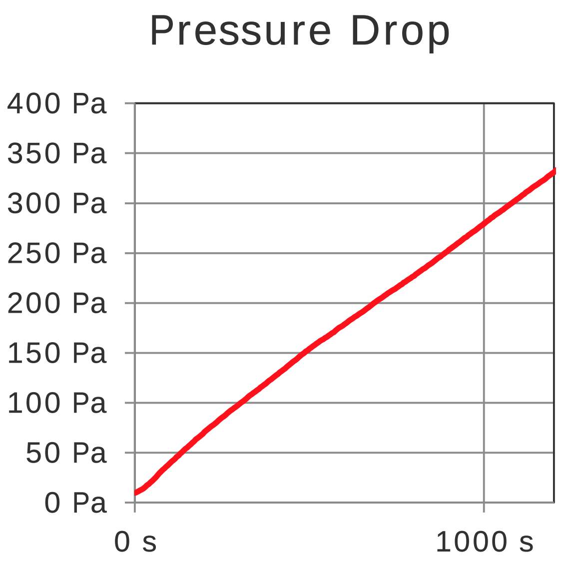




Fully Resolved Particles
Step 1: Caught On First Touch

Resolution 0.5 µm

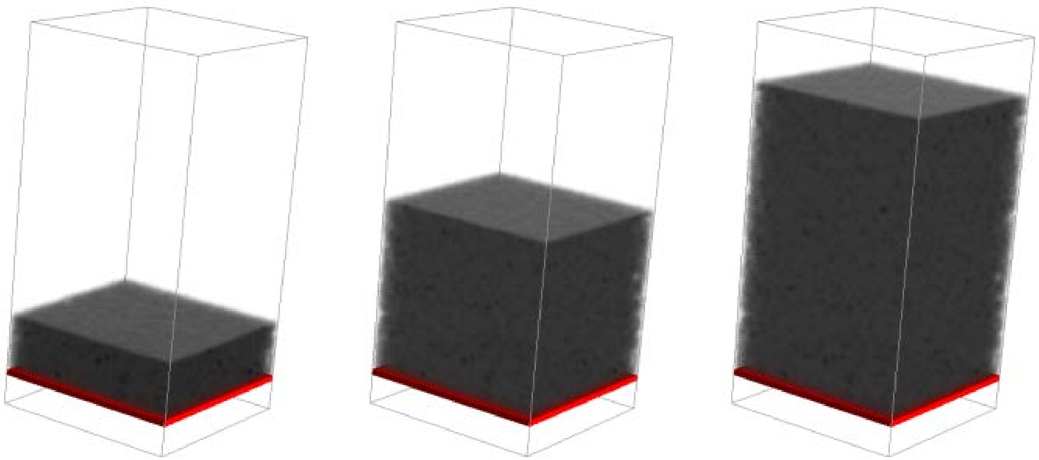


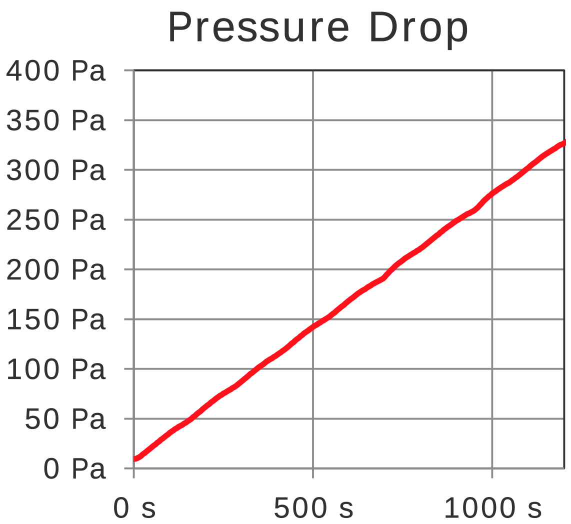


Result:
- Flow resistivity 14.4 e+6 kg/m³s
- Cake solidity 0.1953

Fully Unresolved Particles
Step 1: Caught On First Touch

Resolution 24 µm

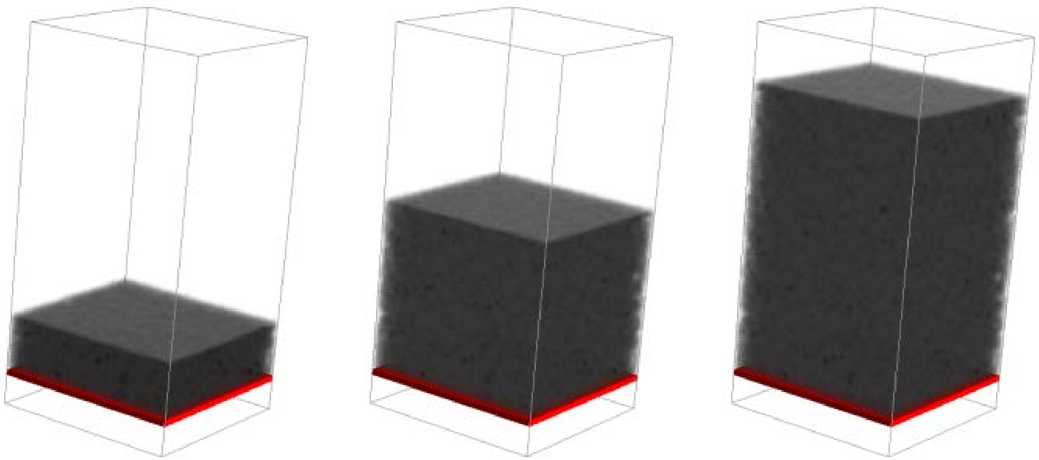


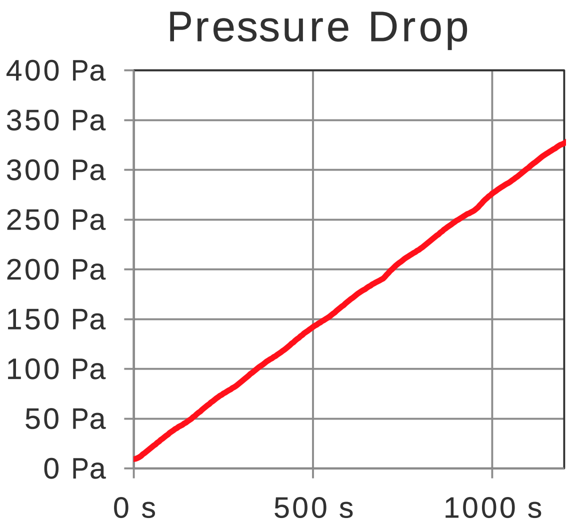


Input (porous voxels):
σ_{max} = 14.4 e+6 kg/m³s
f_{max} = 0.1953

Fully Unresolved Particles
Step 1: Caught On First Touch

Resolution 24 µm





Input (porous voxels):
σ_{max} = 14.4 e+6 kg/m³s
f_{max} = 0.1953

Result:
- Flow resistivity 14.3 e+6 kg/m³s
- Cake solidity 0.2027

Intermediate Resolutions

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^6 kg/m³s]	Solidity	Flow Resistivity [10^6 kg/m³s]
24 µm	0.1953	14.4	0.2027	14.34
8 µm	0.1953	14.4	0.1953	10.17
4 µm	0.1953	14.4	0.1422	4.02
2 µm	0.1953	14.4	0.1346	3.09
1 µm	0.1953	14.4	0.1535	4.41
0.5 µm	solid/empty	solid/empty	0.1953	14.40

Intermediate Resolutions

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^6 kg / m³s]	Solidity	Flow Resistivity [10^6 kg / m³s]
24 µm	0.1953	14.4	0.2027	14.34
8 µm	0.1953	14.4	0.1953	10.17
4 µm	0.1953	14.4	0.1422	4.02
2 µm	0.1953	14.4	0.1346	3.09
1 µm	0.1953	14.4	0.1535	4.41
0.5 µm	solid/empty	solid/empty	0.1953	14.40

Solidity too low

MATH 2 MARKET

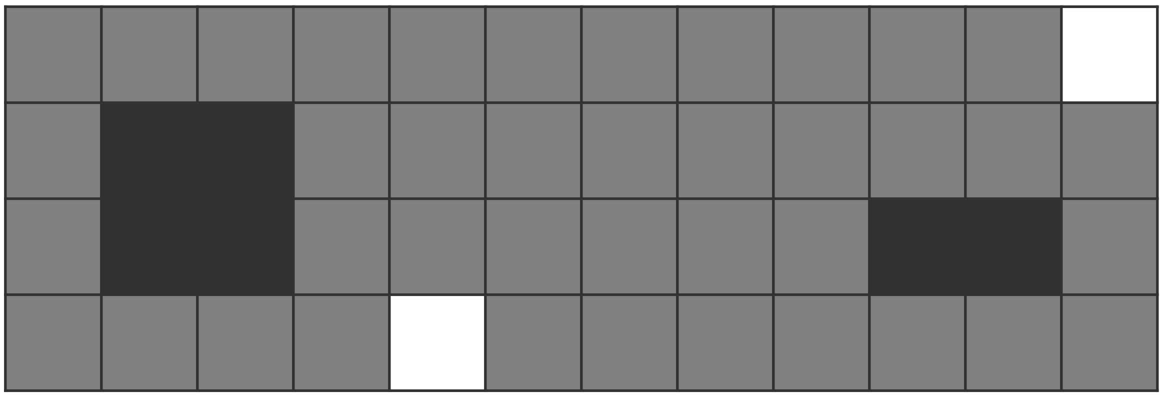
Intermediate Resolutions

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^6 kg / m³s]	Solidity	Flow Resistivity [10^6 kg / m³s]
24 µm	0.1953	14.4	0.2027	14.34
8 µm	0.1953	14.4	0.1953	10.17
4 µm	0.1953	14.4	0.1422	4.02
2 µm	0.1953	14.4	0.1346	3.09
1 µm	0.1953	14.4	0.1535	4.41
0.5 µm	solid/empty	solid/empty	0.1953	14.40

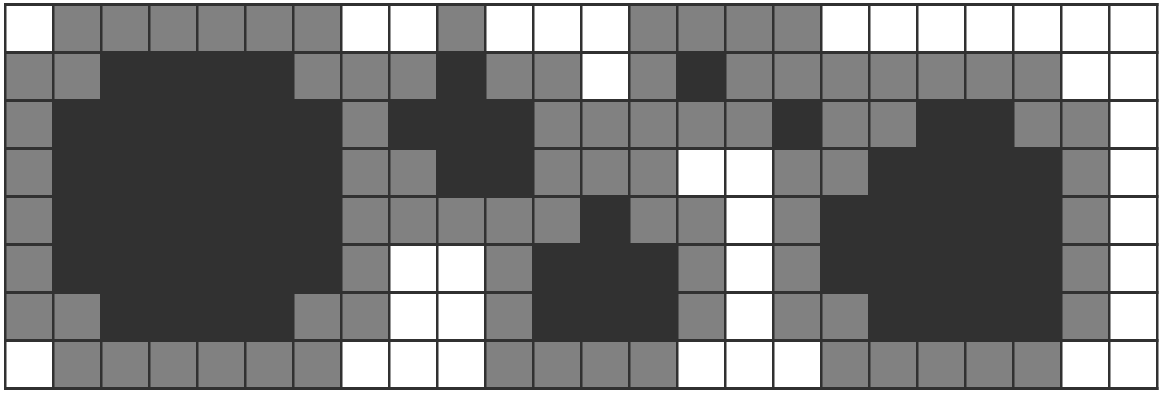
Solidity too low

Resistivity too low

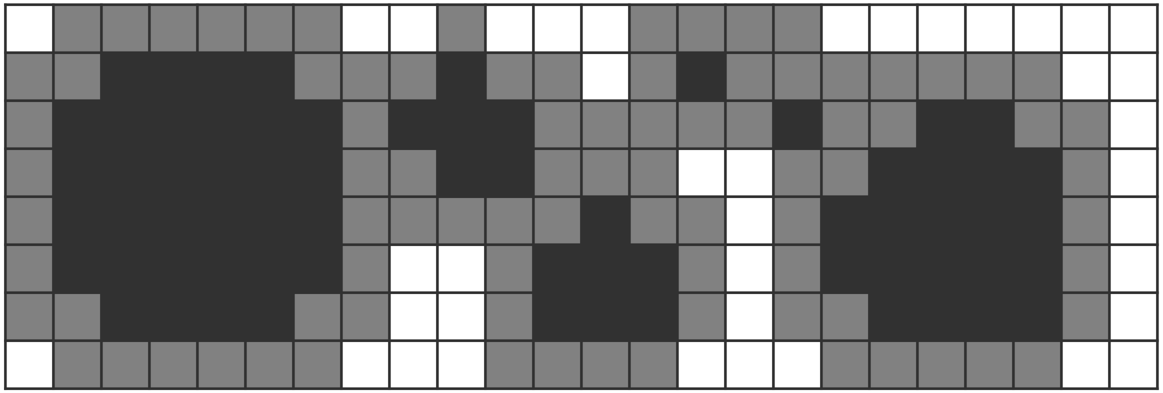
Why?



Why?

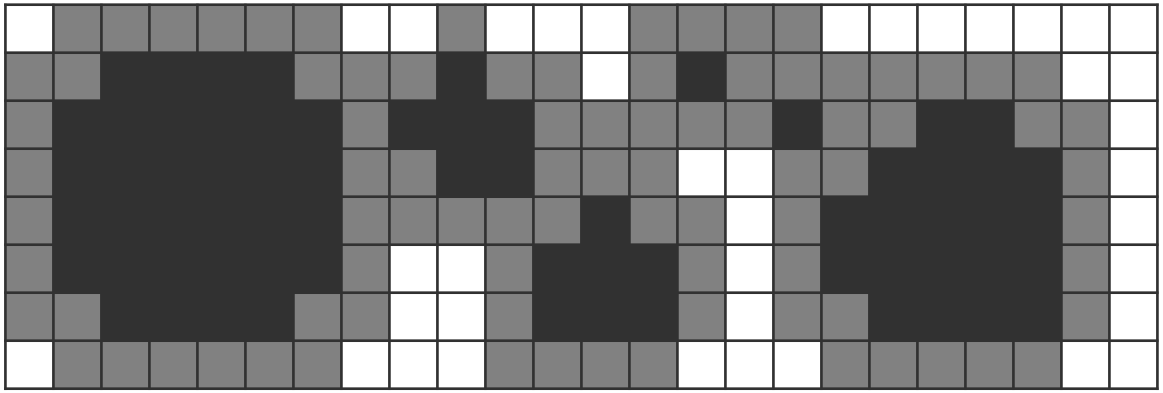


Why?



Reason I:
f_{max} = 0.5 for the solid/empty case

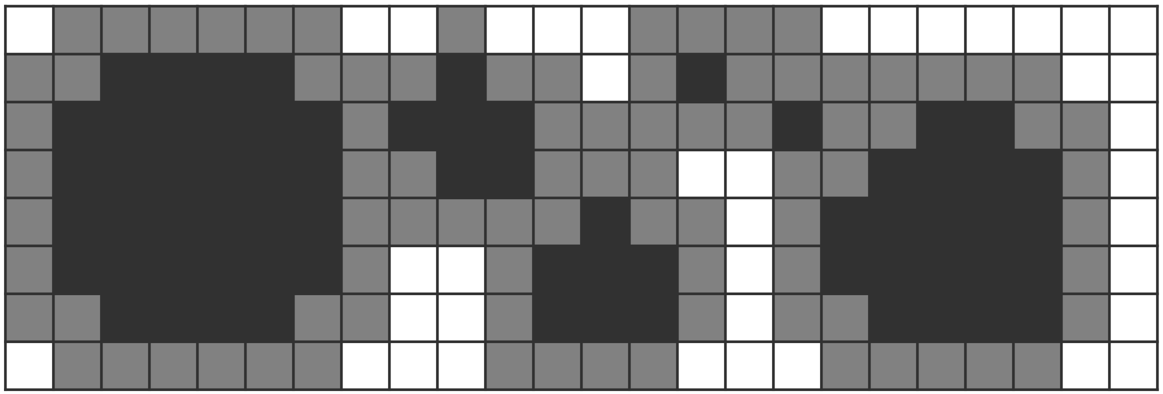
Why?



Reason I:
f_{max} = 0.5 for the solid/empty case

Reason II:
Small voxels do not contain large pores

Why?



Reason I:
f_{max} = 0.5 for the solid/empty case

Reason II:
Small voxels do not contain large pores

=> For higher resolutions:

I. f_{max} should go to 0.5
II. σ_{max} should be increased

Parameter Fitting
Step 1: Caught On First Touch

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^6 kg / m³s]	Solidity	Flow Resistivity [10^6 kg / m³s]
24 µm	0.1953	14.4	0.2027	14.3
8 µm	0.1953	20.19	0.1967	14.4
4 µm	0.2956	56.34	0.1904	13.5
2 µm	0.4600	170.00	0.1949	13.8
1 µm	0.5000	441.50	0.1928	15.2
0.5 µm	solid/empty	solid/empty	0.1953	14.4

MATH 2 MARKET

Quick Parameter Fitting
Step 1: Caught On First Touch

Resolution 4 µm

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^6 kg / m³s]	Solidity	Flow Resistivity [10^6 kg / m³s]
0.1953	14.40	0.1422	4.02
0.4000	200.00	0.2505	50.00
0.2956	56.34	0.1904	13.50

MATH 2 MARKET

Quick Parameter Fitting
Step 1: Caught On First Touch

Resolution 4 µm

1. Use result of resolved model

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^6 kg/m³s]	Solidity	Flow Resistivity [10^6 kg/m³s]
0.1953	14.40	0.1422	4.02
0.4000	200.00	0.2505	50.00
0.2956	56.34	0.1904	13.50

Quick Parameter Fitting
Step 1: Caught On First Touch

Resolution 4 µm

1. Use result of resolved model
2. Use other values

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^6 kg / m³s]	Solidity	Flow Resistivity [10^6 kg / m³s]
0.1953	14.40	0.1422	4.02
0.4000	200.00	0.2505	50.00
0.2956	56.34	0.1904	13.50

Quick Parameter Fitting
Step 1: Caught On First Touch

Resolution 4 µm

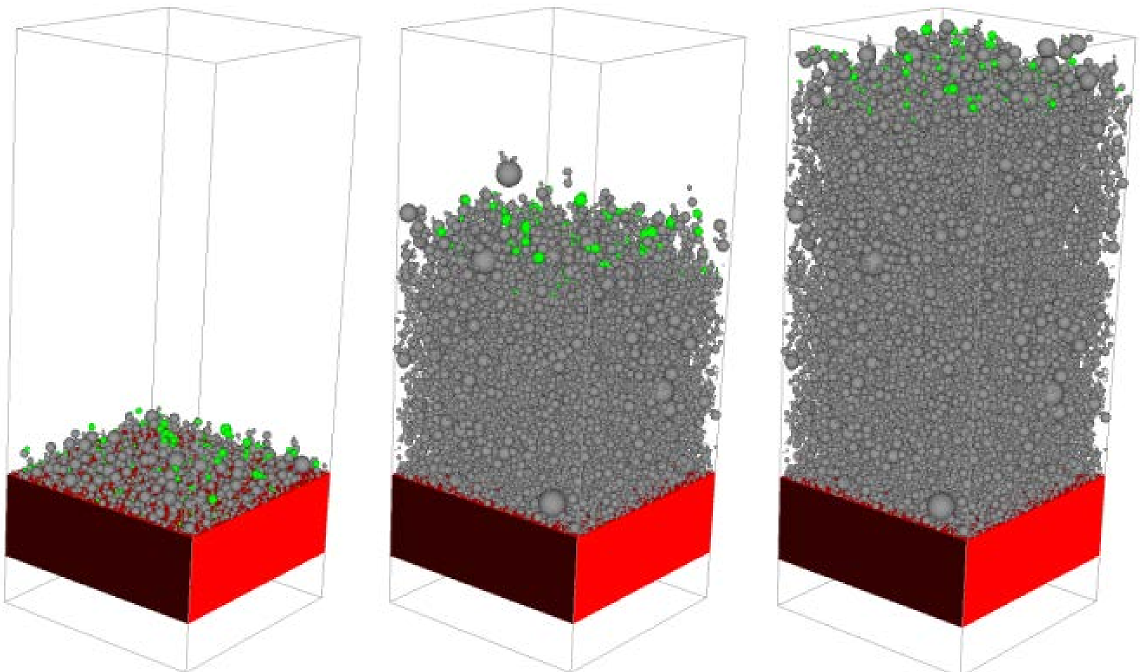
1. Use result of resolved model
2. Use other values
3. Assume linear dependency
   - solidity from f_{max}
   - resistivity from σ_{max}

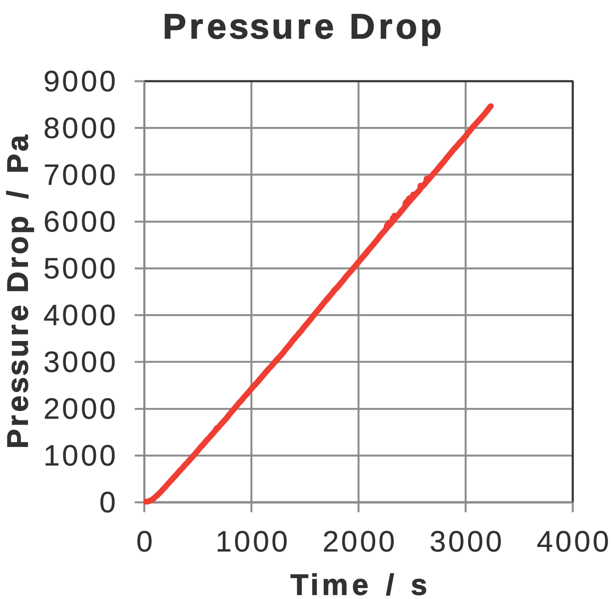
Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^6 kg / m³s]	Solidity	Flow Resistivity [10^6 kg / m³s]
0.1953	14.40	0.1422	4.02
0.4000	200.00	0.2505	50.00
0.2956	56.34	0.1904	13.50

Step 2: Sieving

Fully Resolved Particles
Step 2: Sieving

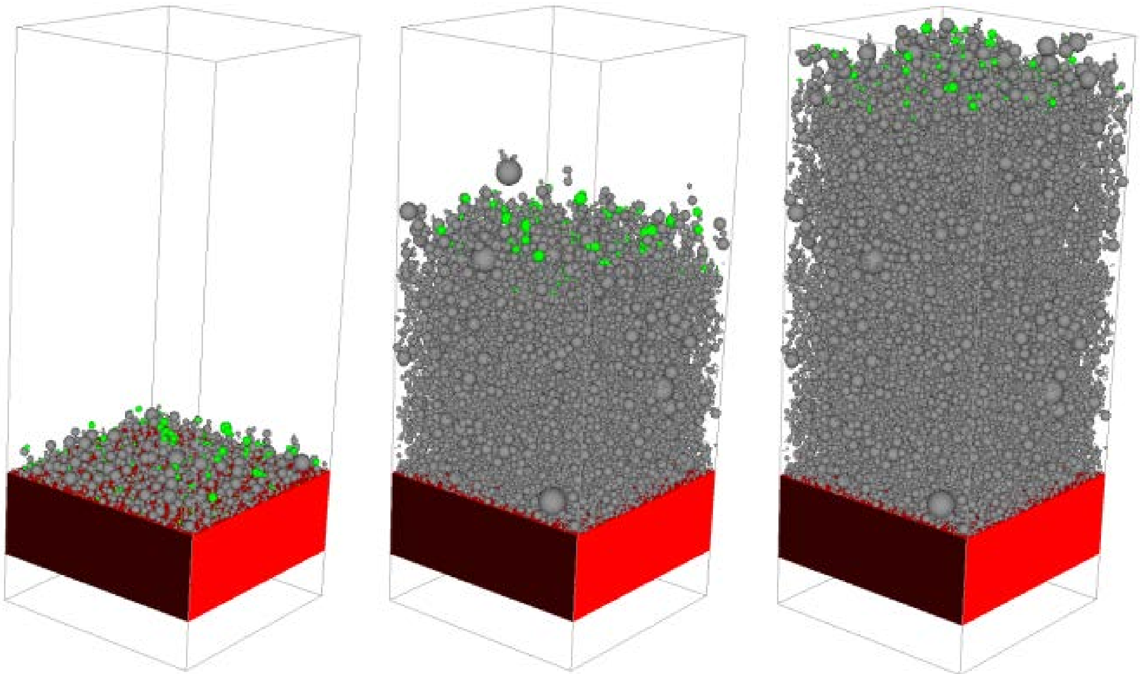
Resolution 0.5 µm

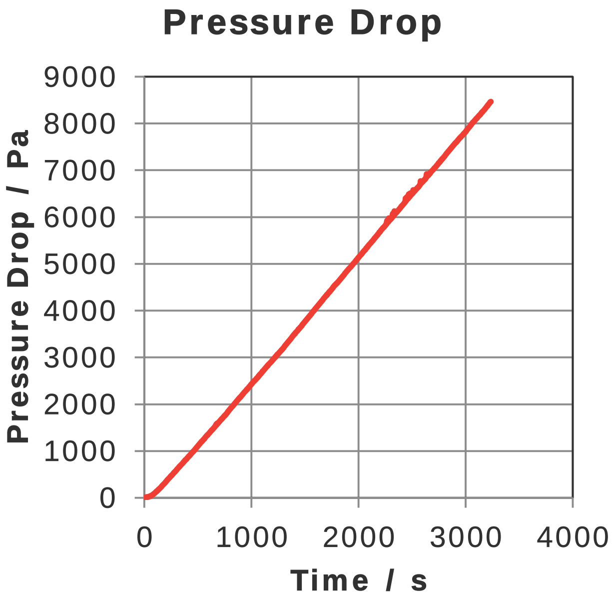




Fully Resolved Particles
Step 2: Sieving

Resolution 0.5 µm



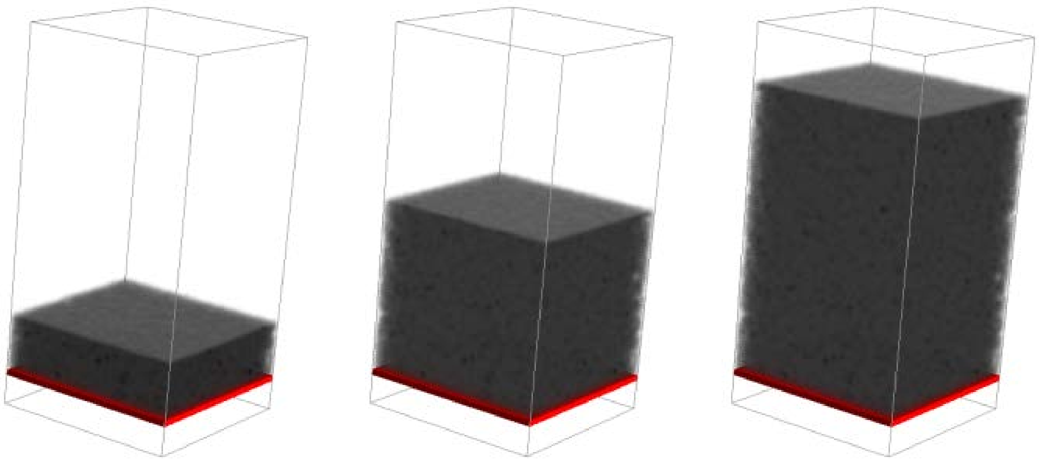


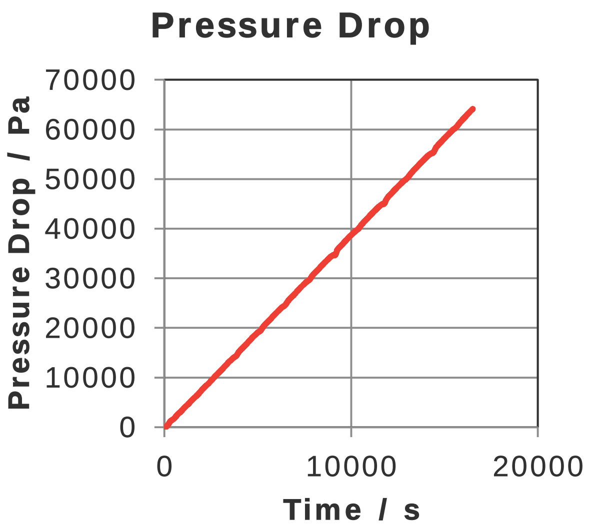
Result:
- Flow resistivity 5.34 e+8 kg/m³s
- Cake solidity 0.4964

MATH 2 MARKET

Fully Unresolved Particles
Step 2: Sieving

Resolution 24 µm





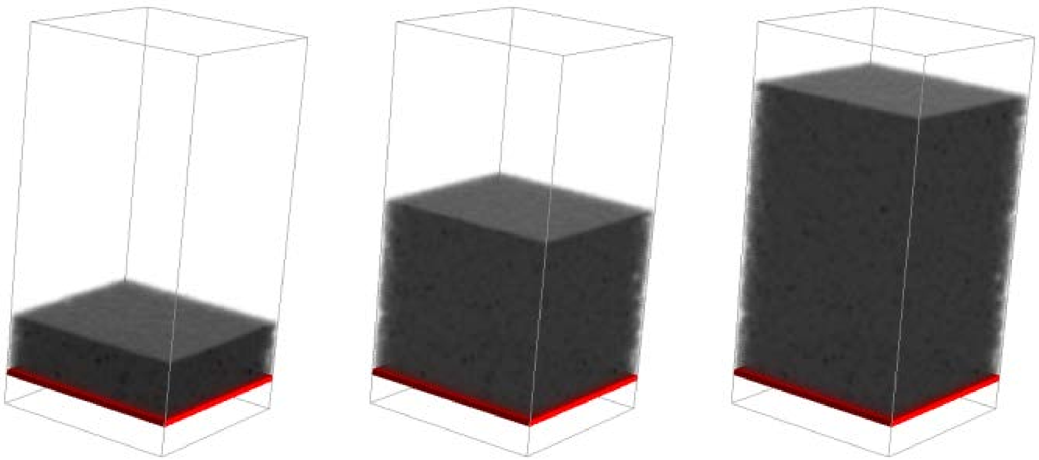
Input (porous voxels):

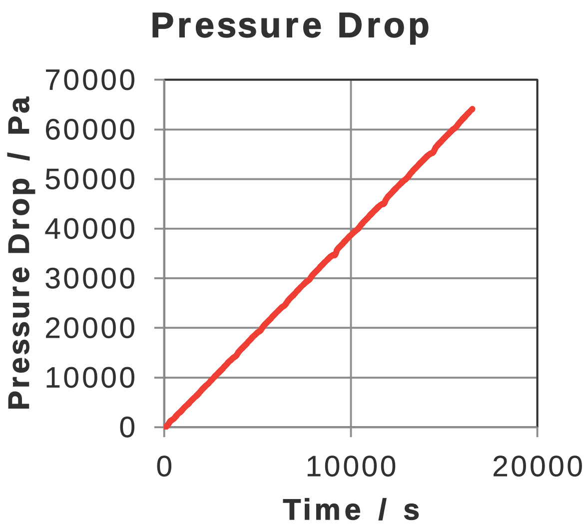
σ_{max} = 5.34 e+8 kg/m³s

f_{max} = 0.4964

Fully Unresolved Particles
Step 2: Sieving

Resolution 24 µm





Input (porous voxels):
σ_{max} = 5.34 e+8 kg/m³s
f_{max} = 0.4964

Result:
- Flow resistivity 5.21 e+8 kg/m³s
- Cake solidity 0.515

Intermediate Resolutions

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^8 kg / m³s]	Solidity	Flow Resistivity [10^8 kg / m³s]
24 µm	0.4964	5.34	0.5150	5.21
8 µm	0.4964	5.34	0.6053	5.47
4 µm	0.4964	5.34	0.6218	5.79
2 µm	0.4964	5.34	0.5343	4.30
1 µm	0.4964	5.34	0.4877	2.64
0.5 µm	solid/empty	solid/empty	0.4964	5.34

Intermediate Resolutions

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^8 kg / m³s]	Solidity	Flow Resistivity [10^8 kg / m³s]
24 µm	0.4964	5.34	0.5150	5.21
8 µm	0.4964	5.34	0.6053	5.47
4 µm	0.4964	5.34	0.6218	5.79
2 µm	0.4964	5.34	0.5343	4.30
1 µm	0.4964	5.34	0.4877	2.64
0.5 µm	solid/empty	solid/empty	0.4964	5.34

Solidity wrong

Intermediate Resolutions

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^8 kg / m³s]	Solidity	Flow Resistivity [10^8 kg / m³s]
24 µm	0.4964	5.34	0.5150	5.21
8 µm	0.4964	5.34	0.6053	5.47
4 µm	0.4964	5.34	0.6218	5.79
2 µm	0.4964	5.34	0.5343	4.30
1 µm	0.4964	5.34	0.4877	2.64
0.5 µm	solid/empty	solid/empty	0.4964	5.34

Solidity wrong

Resistivity wrong

MATH 2 MARKET

Parameter Fitting
Step 2: Sieving

Resolution	Input Parameters		Resulting Cake	
	f_{max}	σ_{max} [10^8 kg/m³s]	Solidity	Flow Resistivity [10^8 kg/m³s]
24 µm	0.4964	5.34	0.5150	5.21
8 µm	0.3820	5.30	0.4940	5.33
4 µm	0.3388	5.39	0.4926	5.33
2 µm	0.4010	7.00	0.4938	5.42
1 µm	0.5100	13.00	0.4919	5.23
0.5 µm	solid/empty	solid/empty	0.4964	5.34

MATH 2 MARKET

Quick Parameter Fitting
Step 2: Sieving

Resolution 4 µm

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^8 kg / m³s]	Solidity	Flow Resistivity [10^8 kg / m³s]
0.4964	5.34	0.6218	5.79
0.6000	50.00	0.7072	61.00
0.3388	5.39	0.4926	5.33

Quick Parameter Fitting
Step 2: Sieving

Resolution 4 µm

1. Use result of resolved model

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^8 kg / m³s]	Solidity	Flow Resistivity [10^8 kg / m³s]
0.4964	5.34	0.6218	5.79
0.6000	50.00	0.7072	61.00
0.3388	5.39	0.4926	5.33

Quick Parameter Fitting
Step 2: Sieving

Resolution 4 µm

1. Use result of resolved model
2. Use other values

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^8 kg / m³s]	Solidity	Flow Resistivity [10^8 kg / m³s]
0.4964	5.34	0.6218	5.79
0.6000	50.00	0.7072	61.00
0.3388	5.39	0.4926	5.33

Quick Parameter Fitting
Step 2: Sieving

Resolution 4 µm

1. Use result of resolved model
2. Use other values
3. Assume linear dependency
   - solidity from f_{max}
   - resistivity from σ_{max}

Input Parameters		Resulting Cake	
f_{max}	σ_{max} [10^8 kg/m³s]	Solidity	Flow Resistivity [10^8 kg/m³s]
0.4964	5.34	0.6218	5.79
0.6000	50.00	0.7072	61.00
0.3388	5.39	0.4926	5.33

MATH 2 MARKET

Conclusions And Outlook

Cake formation can be modeled at different resolutions

- and σ_{max} depend on
  - resolution
  - particle size distribution
  - flow velocity
  - collision model

- Open questions:
  - Does this work for all size distributions / fluids / materials ?
  - Is there a formula for the resolution dependence ?

Conclusions And Outlook

and $\sigma\ max\ \sigma\sigma\ \sigma\ max\ mmaaxx\ \sigma\ max$ depend on

Cake formation can be modeled at different resolutions

- Parameters $f\ max\ \ max\ x\ max$ and σ_{max} depend on
  - resolution
  - particle size distribution
  - flow velocity
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Conclusions And Outlook

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MATH 2 MARKET

Conclusions And Outlook

article size distribution

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Cake formation can be modeled at different resolutions

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Conclusions And Outlook

low velocity

article size distribution

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Conclusions And Outlook

ollision model

low velocity

article size distribution

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Cake formation can be modeled at different resolutions

- resolution
- $cf\ max\ \ max\ x\ max$ and σ_{max} depend on
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MATH 2 MARKET

Conclusions And Outlook

ollision model

low velocity

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Cake formation can be modeled at different resolutions

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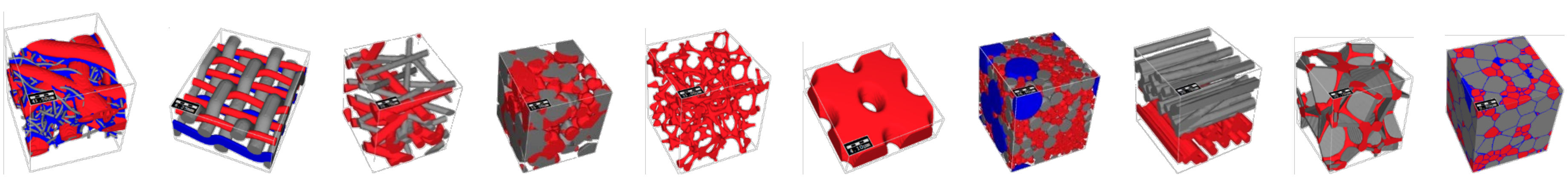
Thank You !

GEO DICT

THE VIRTUAL MATERIAL LABORATORY

Visit us @ www.geodict.com (Table Top 11)



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